Management for Engineers Module 4

Syllabus (Module 4 – Project Management)

- ProjectManagement, Networkconstruction,Arrow diagram, Redundancy. CPMand PERTNetworks, Scheduling computations, PERTtimeestimates, Probabilityof completion of project, Introduction to crashing.

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Project Management

Project Management

- Once a project is selected, the focus is shift toits implementation .
- A project is a seriesof tasks that need to be completedinorder to reach a specific outcome.
- A project can also be defined as a set of activities required to achieve a particular goal.
- The numerous activities(project components) in a project are completed by employing various resources like men, machine, money, and time
- So a project is a set of interconnected/interdependent activities.

Project Management

- The activities ofa project have interrelationshipsarises from physical, technical and other consideration.
- For proper planning, scheduling and control of activities of a project and to completely map the interrelationship among the activities network techniques have been found quite useful.
- The two basic network techniques are PERT and CPM acronym Program Evaluation and Review Technique and Critical Path Method

Project Management

- Widely diverseprojectscanbeanalysedusingPERTand CPM some are listed below:
  - Launching a spaceship
  - Research and development project
  - Construction projects
  - Starting a new venture
- The common characteristics of the projects which are essential for the analysis using PERT and CPM are
  1. Itcanbe brokendown into welldefined sets ofjobsor activities
  2. The activities must be performed in certain sequence
  3. The activities must be start or stopped in independent manner

Project Planning, Scheduling, and Controlling

Project Planning
1. Setting goals
2. Defining the project
3. Tying needs into timed project activities
4. Organizing the team

→ Time/cost estimates
Budgets
Engineering diagrams
Cash flow charts
Material availability details

Project Scheduling
1. Tying resources to specific activities
2. Relating activities to each other
3. Updating and revising on a regular basis

→ CPM/PERT
Gantt charts
Milestone charts
Cash flow schedules

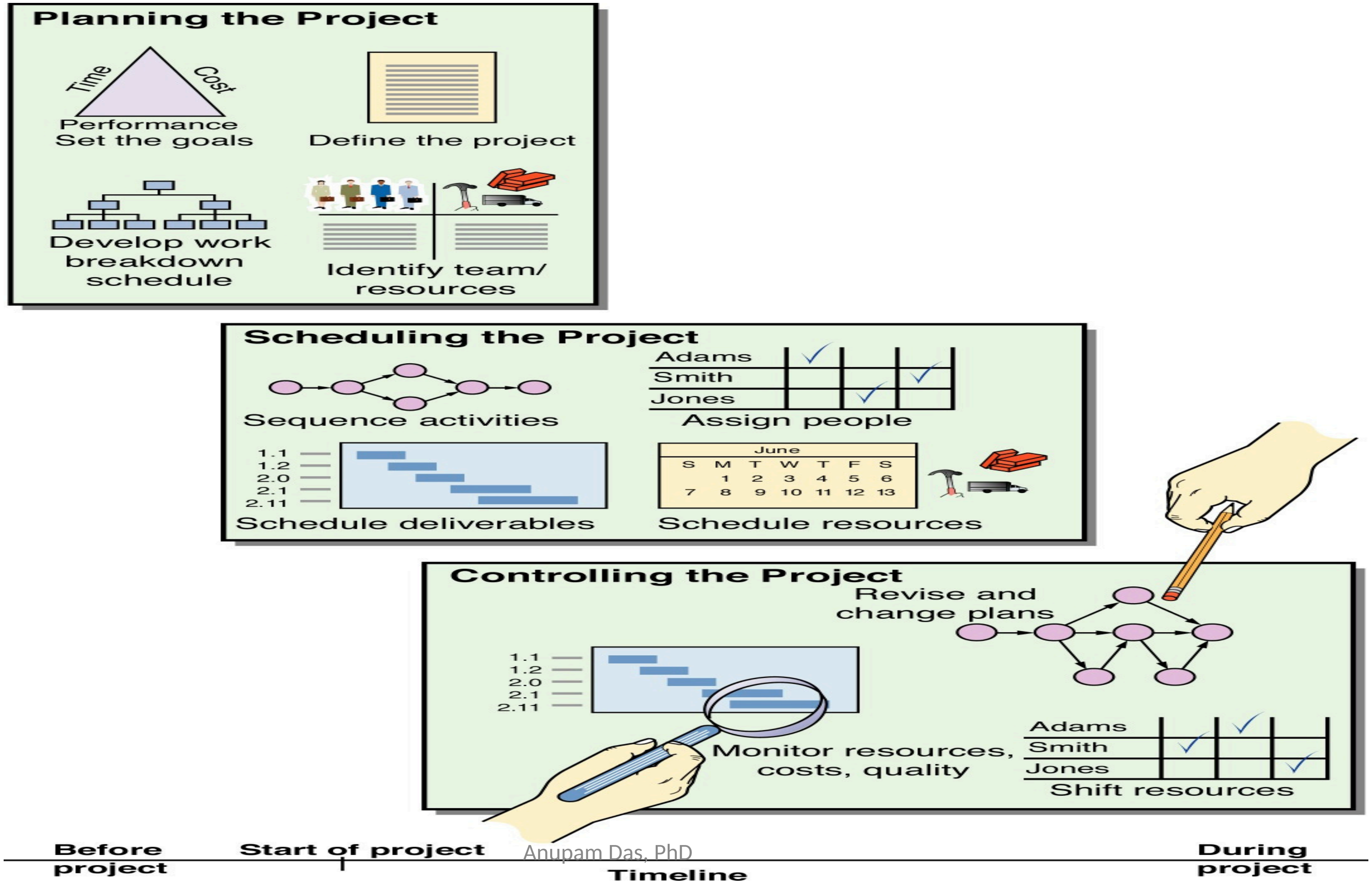
Project Controlling
1. Monitoring resources, costs, quality, and budgets
2. Revising and changing plans
3. Shifting resources to meet demands

→ Reports
- budgets
- delayed activities
- slack activities

Before Project

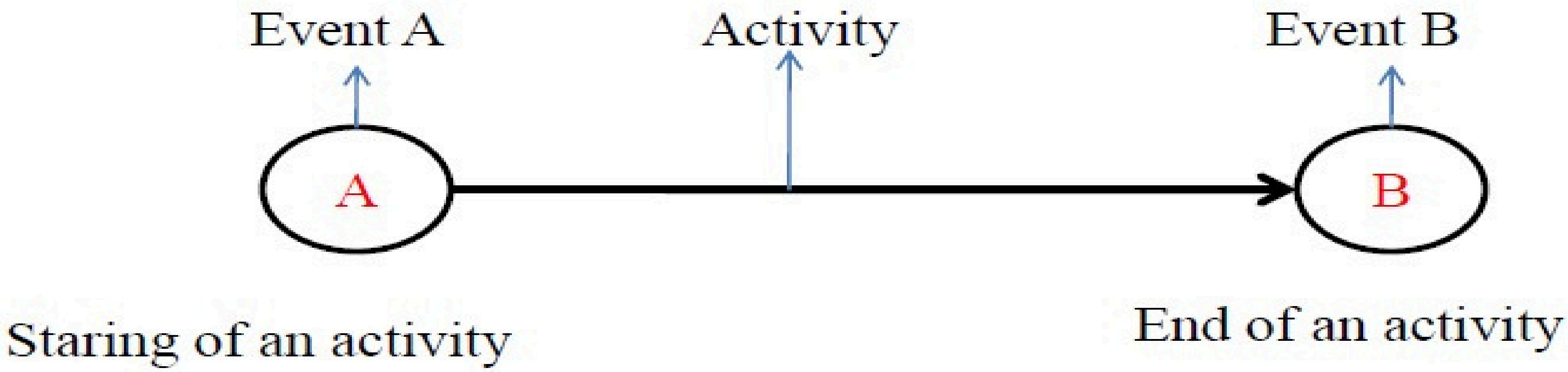
During Project

Project Planning, Scheduling, and Controlling



Network Construction, Arrow Diagram, Redundancy

DEVELOPMENT OF PROJECT NETWORK



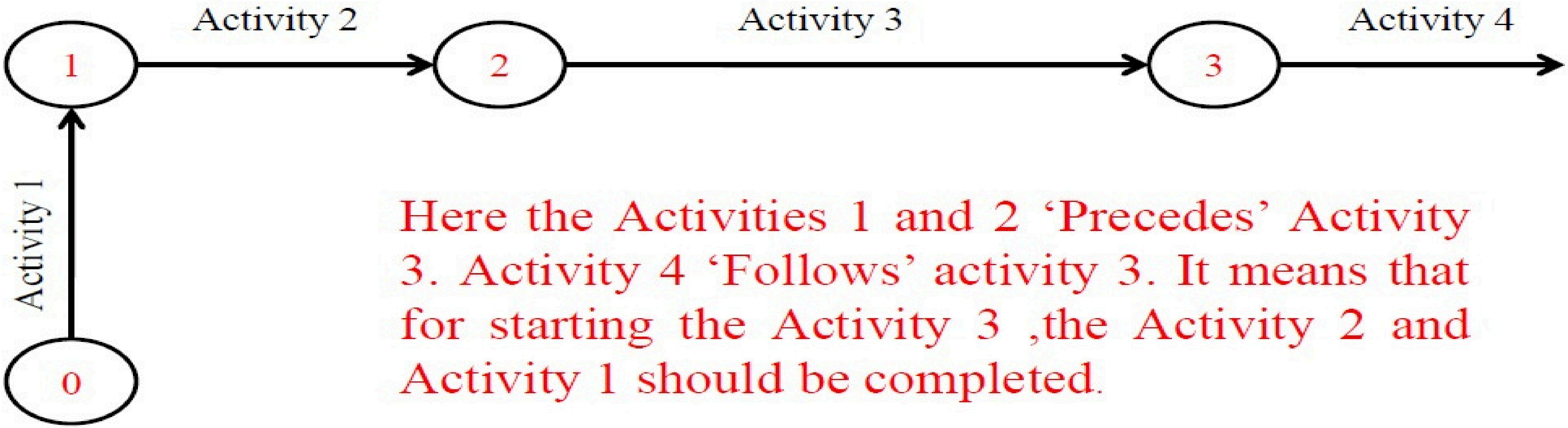
1. The network diagram is constructed in terms of activity and events , An activity is a definite task, job or function to be performed in a project .An activity is represented by arrow . The head of the activity represents the completion of activity and tail of the arrow represents the beginning.
2. An event is a specific point in time indicating the beginning or end of one or more activities. It represents a milestone and does not consume time and resources. Here A and B are events and it is represented by small circle or node

Development of Project Network

- Activityshould be definedthat they are distinct, reasonably homogeneous task for which time and resources requirements can be estimated.
- Activities are building blocks of a project. Once the activities are defined it is necessary to define for each activity ie., the activities which precedes it and activities which follows it.

DEVELOPMENT OF PROJECT NETWORK

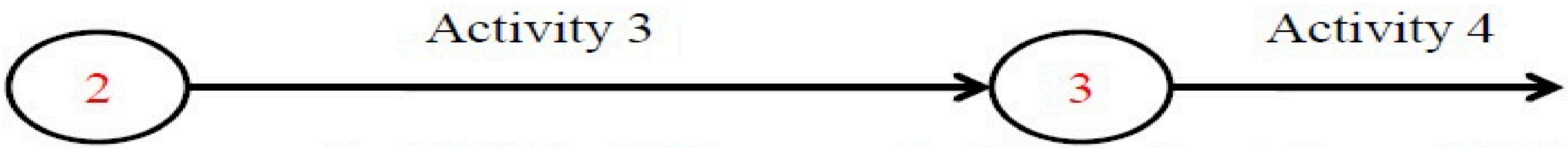
Example:



Here the Activities 1 and 2 'Precedes' Activity 3. Activity 4 'Follows' activity 3. It means that for starting the Activity 3 ,the Activity 2 and Activity 1 should be completed.

RULES FOR CONSTRUCTING THE NETWORK

- Each activity must have a preceding and succeeding event

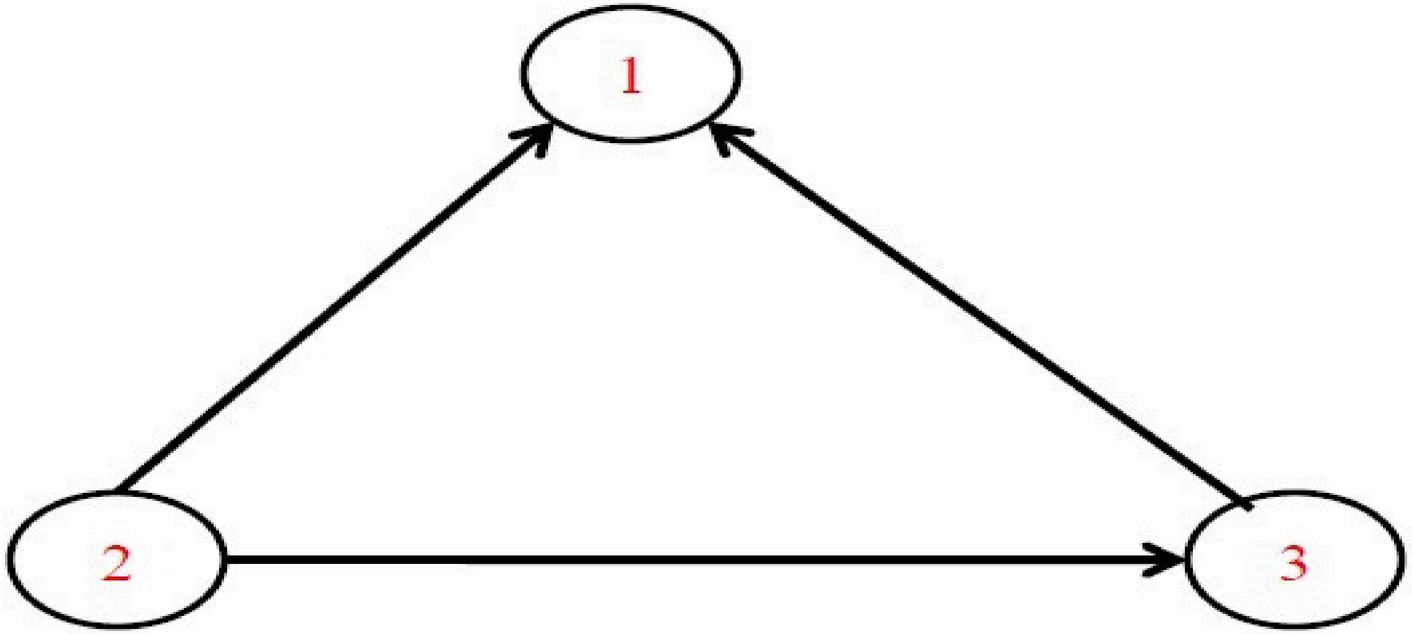


For example Activity 3 is numerically designated as 2-3 here the preceding event is 2 and succeeding is 3

- Every event should be a distinct number, In practice events are so numbered that the number at the head of the arrow is greater than that at its tail

RULES FOR CONSTRUCTING THE NETWORK

- There should not be no loops in the project

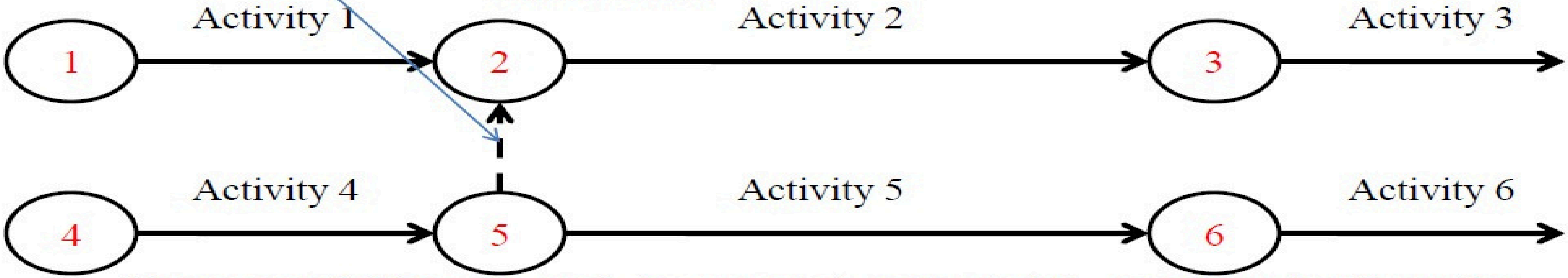


- This situation is not permissible in the network

RULES FOR CONSTRUCTING THE NETWORK

Not more than one activity can have the same preceding and succeeding event

A dummy activity is an imaginary activity which has zero duration and does not consumes resources. It is represented by dashed arrow



Dummy activities are used to represent a constraint, necessary to show the relationships between activities .In the above figure dummy activity is represented by dashed arrow. In the figure dummy activity shows a logical relation , since activity 1 and activity 4 should be completed before activity 2 and activity 5

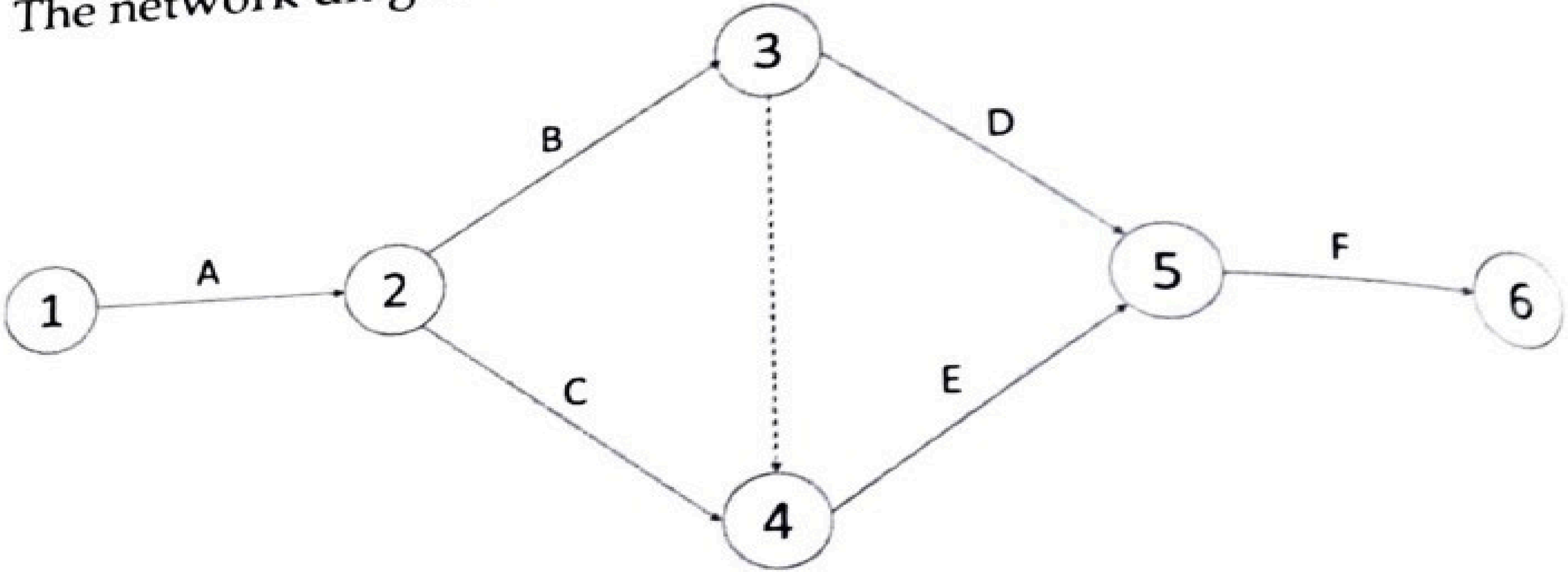
Practice Problem

- A project consists of six activities (jobs) designated from A to F, with the following relationships.Draw the network diagram

1. A is the first job to be performed
2. B and C can be done concurrently and must follow A
3. B must precede D.
4. E must succeed C but it cannot start until B is completed.
5. The last operation F is dependent on the completion of D and E.

Final Answer

The network diagram is



Here 3-4 is a dummy activity

PRACTICE PROBLEM

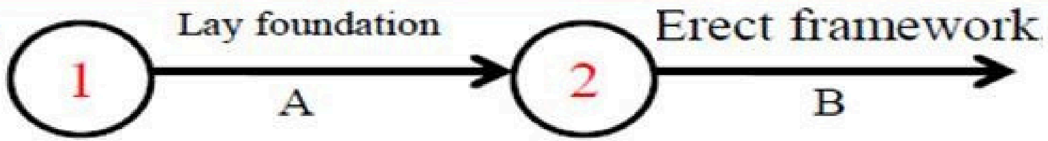
- A project consist of following activities construct the network

  Activities for building project

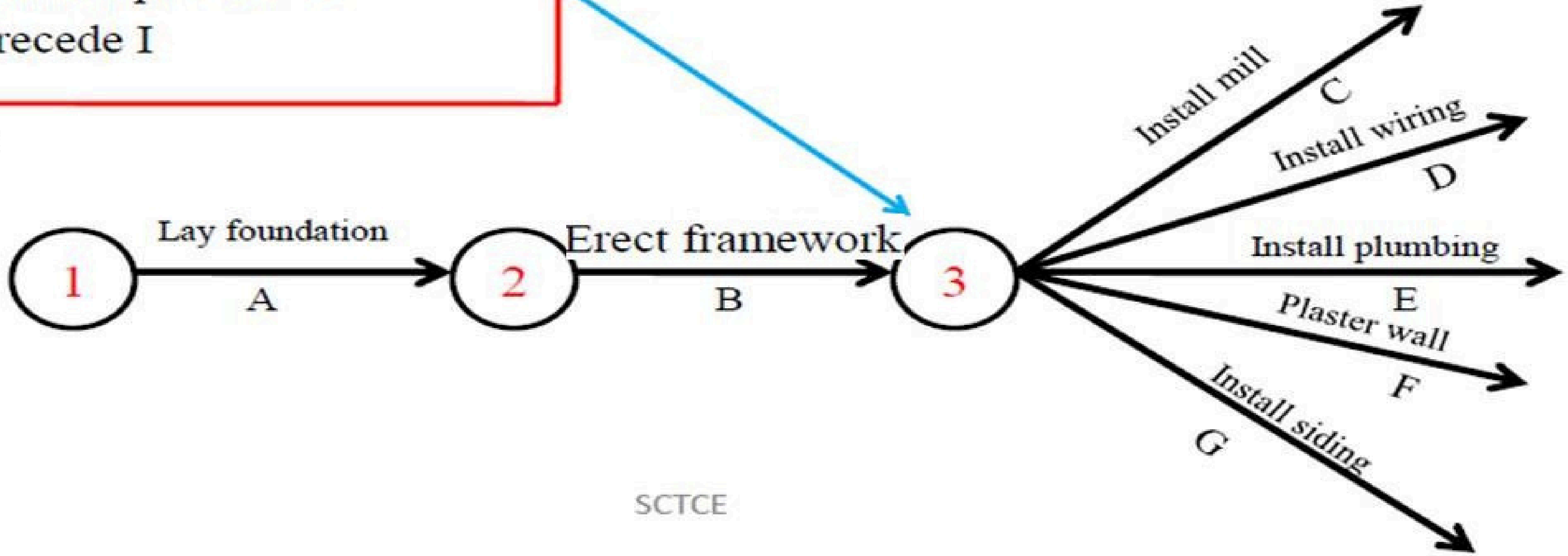
  A= lay foundation , B= Erect framework, C= Install mill work, D= Install wiring , E= Install plumbing, F=plaster wall, G=Install siding, H=Decorate the interior, I= Finish the exterior

The interrelation among these activities are given below

1. A should precede B,
2. B should precede C, D, E, F and G
3. C, D, E, F should precede H
4. G should precede I

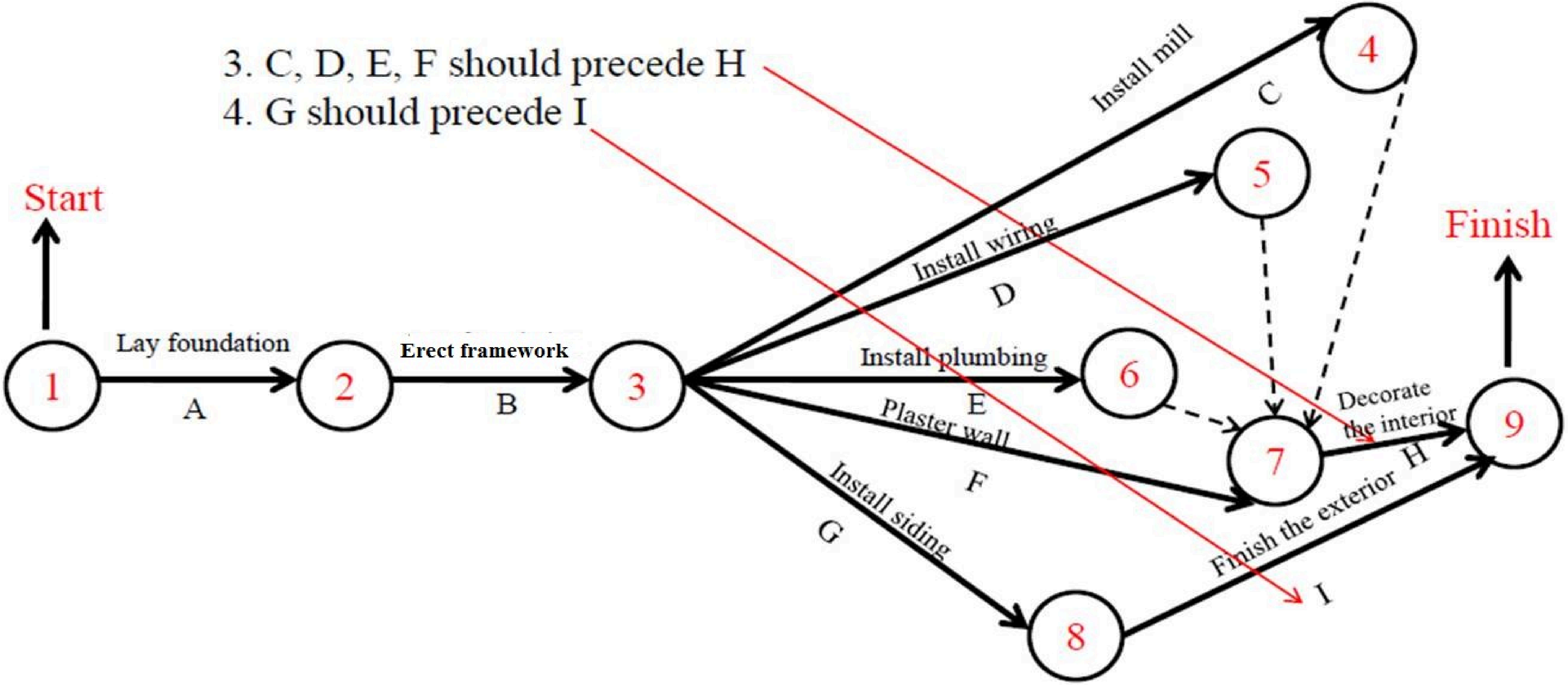


Solution:



FINAL ANSWER

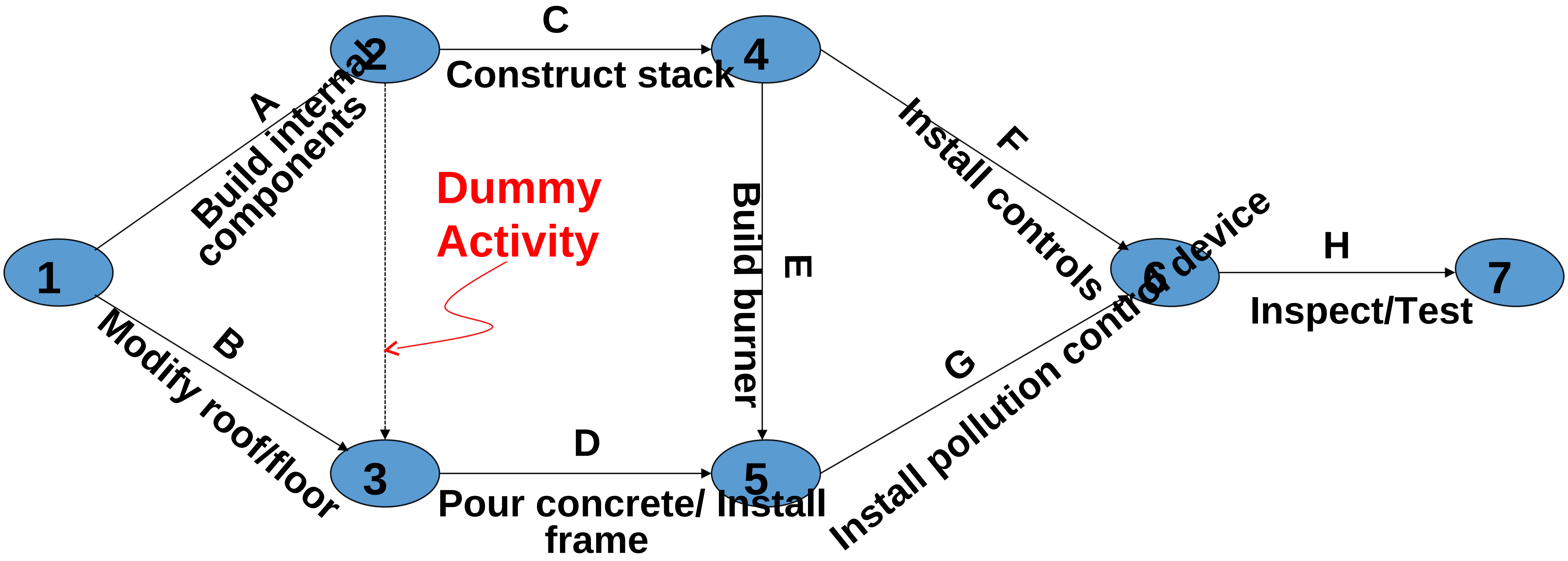
3. C, D, E, F should precede H
4. G should precede I



Practice Problem

Activity	Description	Immediate Predecessors
A	Build internal components	-
B	Modify roof and floor	-
C	Construct collection stack	A
D	Pour concrete and install frame	A, B
E	Build high-temperature burner	C
F	Install pollution control system	C
G	Install air pollution device	D, E
H	Inspect and test	F, G

Final Answer



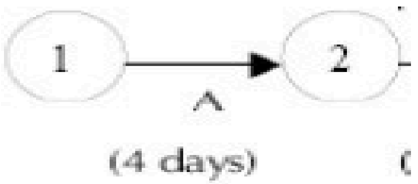
Assignment 1

Table 8.1: Sequence of Activities for House Construction Project

Name of the activity	Starting and finishing event	Description of activity	Predecessor	Time duration (days)
A	(1,2)	Prepare the house plan	--	4
B	(2,3)	Construct the house	A	58
C	(3,4)	Fix the door / windows	B	2
D	(3,5)	Wiring the house	B	2
E	(4,6)	Paint the house	C	1
F	(5,6)	Polish the doors / windows	D	1

Construct the network

Hint : please represent duration as given in the figure



Assignment 2

Example 2: Consider the project given in Table 8.2 and construct a network diagram. **Table 8.2: Sequence of Activities for Building Construction Project**

Activity	Description	Predecessor
A	Purchase of Land	-
B	Preparation of building plan	-
C	Level or clean the land	A
D	Register and get approval	A, B
E	Construct the building	C
F	Paint the building	D

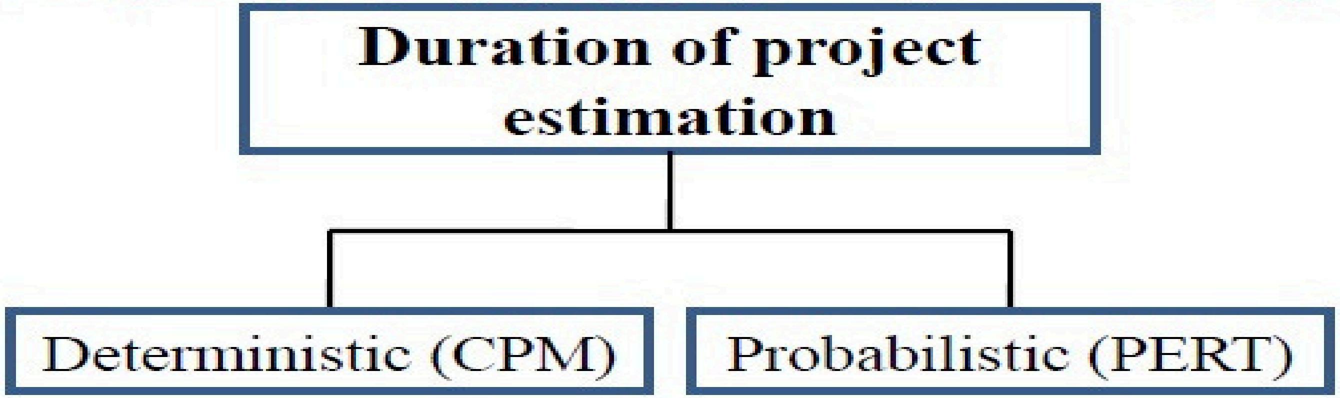
CPM and PERT Networks, Scheduling computations, PERT time estimates, Probability of completion of project

Duration of Project

- The important characteristic of any project isits duration.
- The project duration depends on the time taken for each events to occur
- Duration is the total time that it takes to complete a project measured in work days, hours or weeks.
- The duration depends on the availability and capacity of resources. Effort is the number of people hours needed to complete a task, i.e. it's the actual time that is spent on working on the project.
- The estimate of time duration can done in two ways one is the deterministic approach in which the project manager has enough knowledge project and is capable of giving a single estimate.

DURATION OF PROJECT

- The other is probabilistic approach towards the time duration in which the accurate duration is not available and also difficult to judge the duration.
- In the probabilistic approach only the limits within which the duration will lie is estimated.



DURATION OF PROJECT

- In the CPM the duration of each activities is assed by the planner who is having enough knowledge about the project, and gives a single estimate of the duration of activities

- The PERT **estimate** (t_e) is based on a formula that includes your optimistic **time estimate** (t_o), your most likely **time estimate** (t_m) and your pessimistic **time estimate** (t_p).

The basic equation is this: $t_e = \frac{(t_o + 4\,t_m + t_p)}{6}$

Optimistic time t_o :It is the estimate of minimum time required for an activity if every thing goes as per the ideal conditions ie if the ideal condition prevails.

DURATION OF PROJECT

Pessimistic time t_p :It is an estimate of the maximum time required for an activity if bad luck is experienced. It may take into account an initial failure or delay but should be influenced by major hazard (such as floods or earthquakes) unless this are inherent in the activity

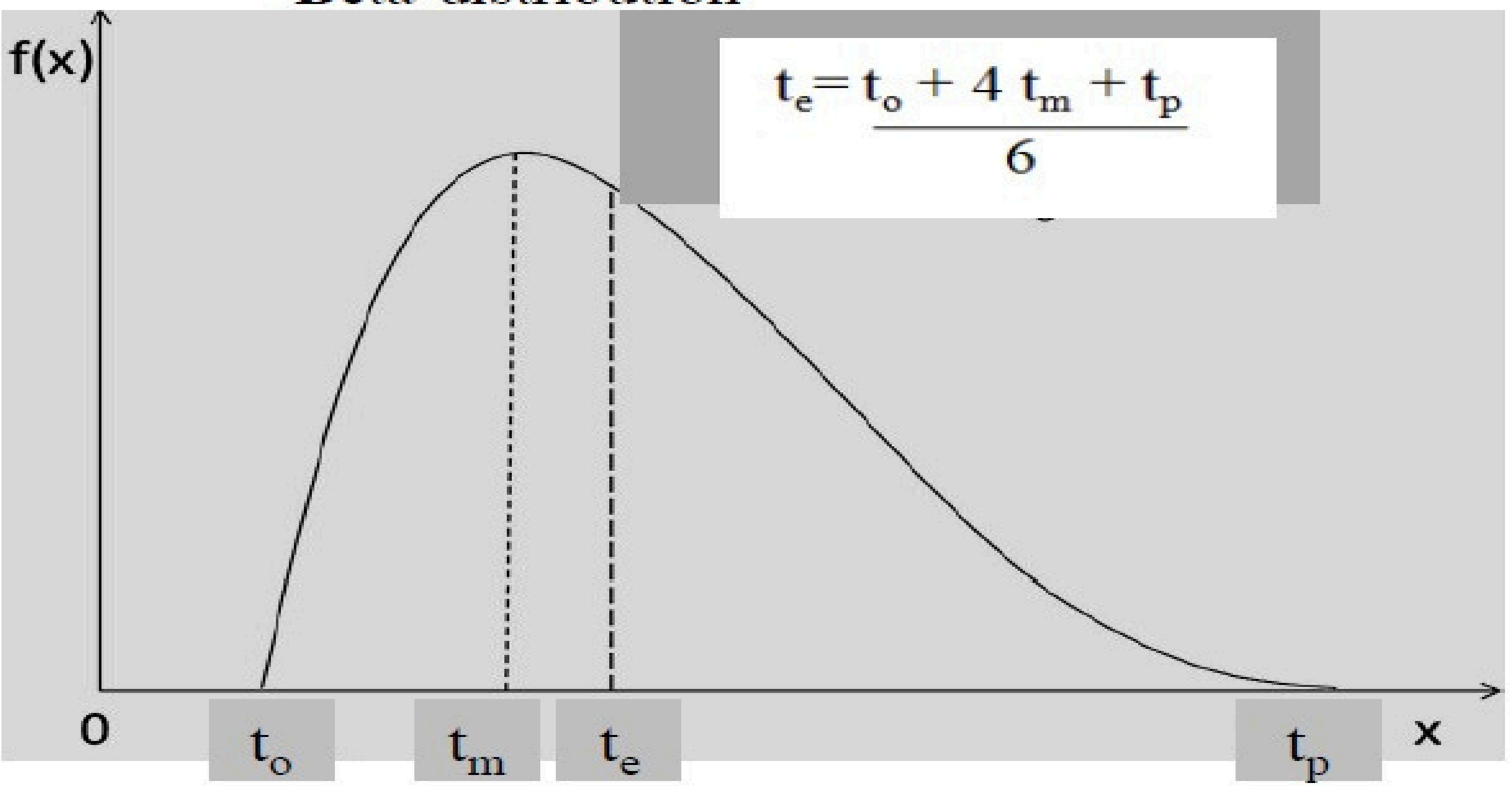
Most likely time t_m It is based on experience and judgement being the time required if normal condition prevail. It lies between Optimistic and pessimistic time

From the above three different estimates, PERT suggests work out of the expected time, expressed as 't_e' assuming that the probability distribution of the activity duration follows beta-distribution and, thus, t_e is the average of t_o t_p and t_m calculated as,

$$t_e = \frac{t_o + 4\,t_m + t_p}{6}$$

TIME ESTIMATE /DURATION

Beta-distribution



$$t_e = \frac{t_o + 4\,t_m + t_p}{6}$$

The general shape of the Beta-distribution is shown in the figure. It is seen that the peak of the curve or the mode corresponds to the most likely time $t_{m.}$ This peak may take up any position within the range of distribution to conform to the characteristics of the activity under consideration.

The range of the distribution roughly determines /define the optimistic and pessimistic times .Because these time estimates represents the extreme cases having little chance of occurrences therefore having very little probability. In Beta distribution a simple approximation can be made for obtaining the activity mean time

TIME ESTIMATE /DURATION

The expected mean time is obtained from the equation $t_e = \frac{t_o + 4\,t_m + t_p}{6}$

The 3 estimates of time are such that

$$t_o \le t_m \le t_p$$

Therefore the range for the time estimate is $t_p - t_o$

The time taken by an activity in a project network follows a distribution with a standard deviation of one sixth of the range, approximately.

i.e., The standard deviation = $\sigma = \frac{t_p - t_o}{6}$

and the variance = $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$

Practice problem

From the data given in the Table construct the network and find the expected mean time for each activity, variance and standard deviation

	Duration in days		
Activity	t_o	t_m	t_p
1-2	3	5	7
1-3	4	8	12
1-4	6	8	14
2-4	2	4	7
3-4	4	6	8

Solution:

Sample calculation activity (1-2): t_e = 3+4x5=7 = 5

Practice problem

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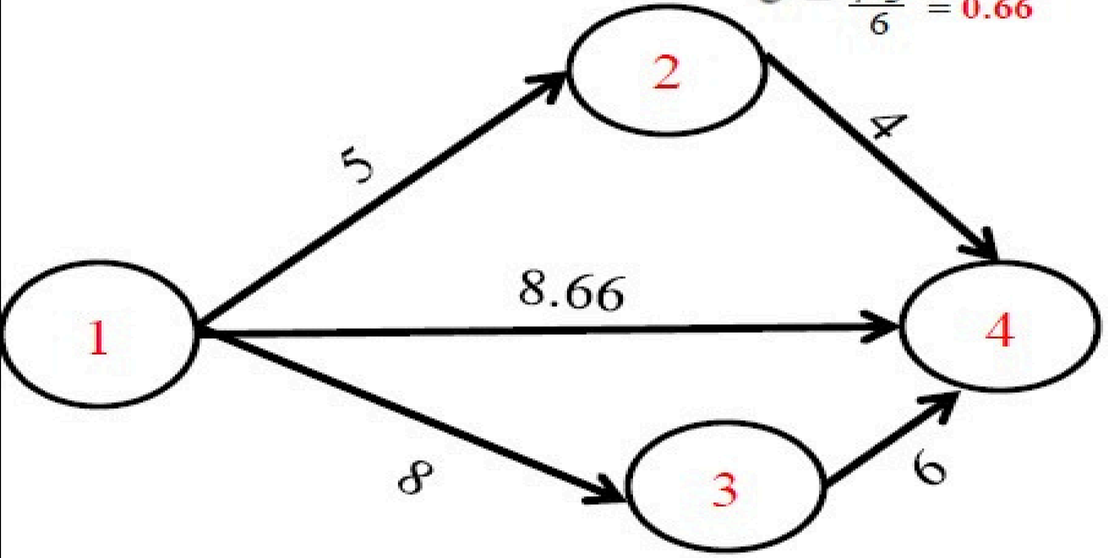
1-3	4	8	12
1-4	6	8	14
2-4	2	4	7
3-4	4	6	8

Solution:

Sample calculation activity (1-2): $t_e = \frac{3+4x5=7}{6} = 5$

$\sigma = \frac{7-3}{6} = 0.66$ $\quad \sigma^2 = (0.66)^2 = .43$



Activity	t_o	t_m	t_p	$t_e = \frac{t_o + 4t_m + t_p}{6}$	$\sigma = \frac{t_p - t_o}{6}$	σ^2
1-2	3	5	7	5	0.66	0.43
1-3	4	8	12	8	1.33	1.76
1-4	6	8	14	8.66	1.33	1.76
2-4	2	4	7	4.166	0.83	0.68
3-4	4	6	8	6	0.66	0.43

Assignment 3

An R & D project has a list of tasks to be performed whose time estimates are given in the Table 8.11, as follows.

Table 8.11: Time Estimates for R & D Project

Activity i j	Activity Name	T_0	t_m (in days)	t_p
1-2	A	4	6	8
1-3	B	2	3	10
1-4	C	6	8	16
2-4	D	1	2	3
3-4	E	6	7	8
3-5	F	6	7	14
4-6	G	3	5	7
4-7	H	4	11	12
5-7	I	2	4	6
6-7	J	2	9	10

Draw the network and find the expected mean time, variance and standard deviation

PROGRAM EVALUATION AND REVIEW TECHNIQUE (PERT)

PROGRAM EVALUATION AND REVIEW TECHNIQUE (PERT)

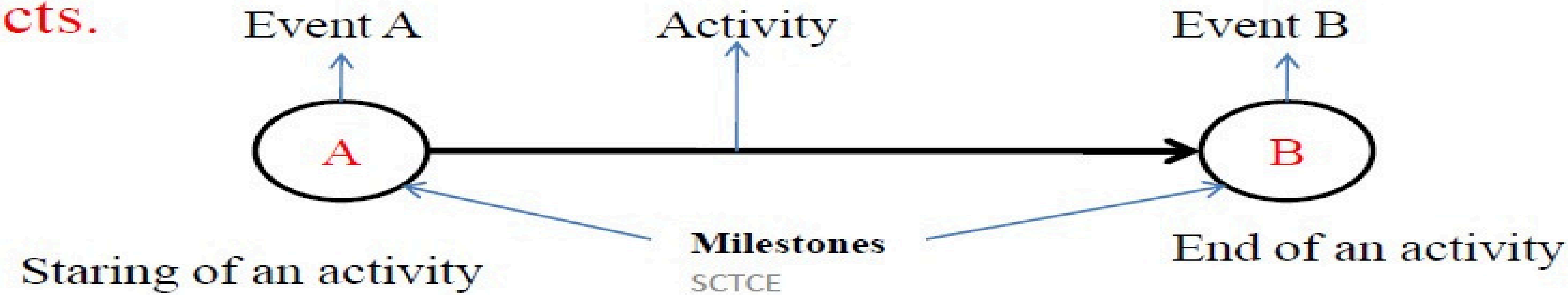
- PERT stands forProgramEvaluationReview Technique, a methodology developed by the U.S. Navy in the 1950s to manage the Polaris submarine missile program.
- A PERT chart, sometimes called a PERT diagram, is a project management tool used to schedule, organize and coordinate tasks within a project

Steps involved in PERT

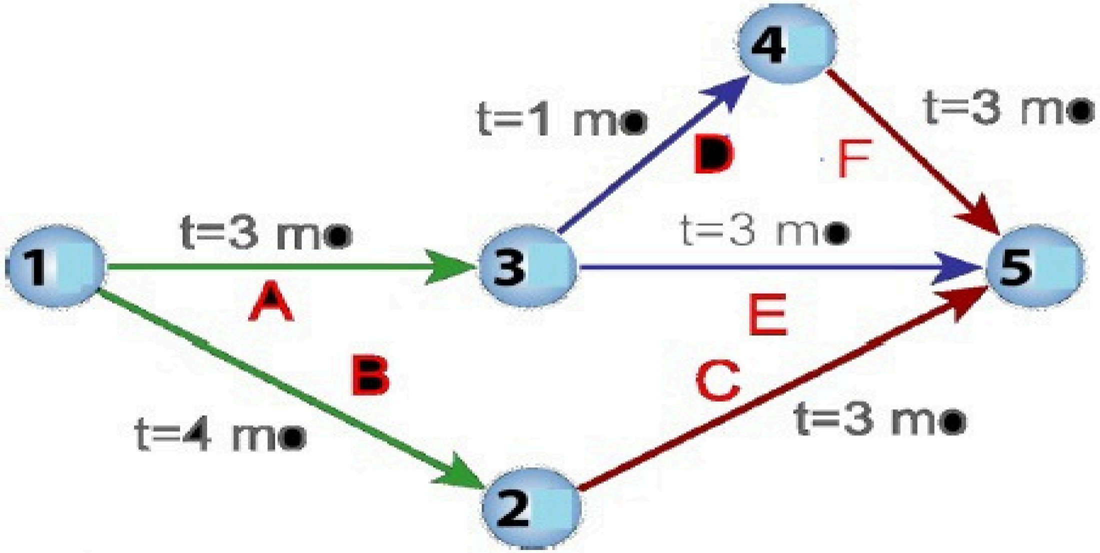
1. Identify the specific activities and milestones.
2. Determine the proper sequence of the activities.
3. Construct a network diagram.
4. Estimate the time required for each activity.
5. Determine the critical path.
6. Update the **PERT chart as the project progresses.**

IDENTIFY THE SPECIFIC ACTIVITIES AND MILESTONES

- The activities are tasks to complete the project
- The milestones are the events marking the beginning and end of one or more activities
- It is more of an event-oriented technique rather than start- and completion-oriented, and is used more in these projects where time is the major factor rather than cost.
- It is applied on very large-scale, one-time, complex, non-routine infrastructure and on Research and Development projects.



DETERMINE THE ACTIVITY SEQUENCE AND CONSTRUCT THE NETWORK DIAGRAM



- Activity sequence is the order in which the activities are performed.
- A network diagram can be drawn from the sequence of series and parallel activities
- In network model the activities are depicted by arrows and the milestones (events) are depicted by circle or bubbles in the network diagram

ESTIMATE THE TIME REQUIRED FOR EACH ACTIVITY

- **The activity time completion units may be in days, weeks, month/year generally weeks are the common unit. The common property of PERT is the ability to deal with uncertainty in the activity completion time .**

- The PERT **estimate** (t_e) is based on a formula that includes your optimistic **time estimate** (t_o), your most likely **time estimate** (t_m) and your pessimistic **time estimate** (t_p).

The basic equation is this: $t_e = \frac{(t_o + 4\ t_m + t_p)}{6}$

Optimistic time t_o :It is the estimate of minimum time required for an activity if every thing goes as per the ideal conditions ie if the ideal condition prevails.

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Most likely time t_m It is based on experience and judgement being the time required if normal condition prevail. It lies between Optimistic and pessimistic time

From the above three different estimates, PERT suggests work out of the expected time, expressed as 't_e' assuming that the probability distribution of the activity duration follows beta-distribution and, thus, t_e is the average of t_o t_p and t_m calculated as,

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The expected mean time is obtained from the equation $t_e = \frac{t_o + 4\,t_m + t_p}{6}$

The 3 estimates of time are such that

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Therefore the range for the time estimate is $t_p - t_o$

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i.e., The standard deviation = $\sigma = \frac{t_p - t_o}{6}$

and the variance = $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$

DETERMINING THE CRITICAL PATH

- The critical path is the longest path (time-wise) in the project network.
- The activities in the **critical path have an effect on the deadline of the project. If an activity of this path is delayed, the project will be delayed.**
- Critical path allows teams to identify the most important tasks in a project.

  This provides a higher level of insight into your project's timeline and a correlation between tasks, giving you more understanding about which task durations you can modify, and which must stay the same
- The **critical path is the longest path that is possible with your planned activities, then the decision maker can figures out the time constraints that each activity is under.**

STEPS IN DETERMINING THE CRITICAL PATH IN PERT

- Step1:Specify EachActivity.
- Step2:Establish Dependencies(ActivitySequence)
- Step3:Draw the Network Diagram.
- Step4:Estimate Activity Completion Time(expected mean time
- Step5:Estimate the occurrence time for each event and compute the slack
- Step6:Identify the Critical Path.
- Step7:Update the Critical Path Diagram to Show Progress.

STEPS IN DETERMINING THE CRITICAL PATH IN PERT

- We have already discussed steps 1-4 (Ref previous slides)
- Step 5: Estimate the occurrence time for each event and compute the slack

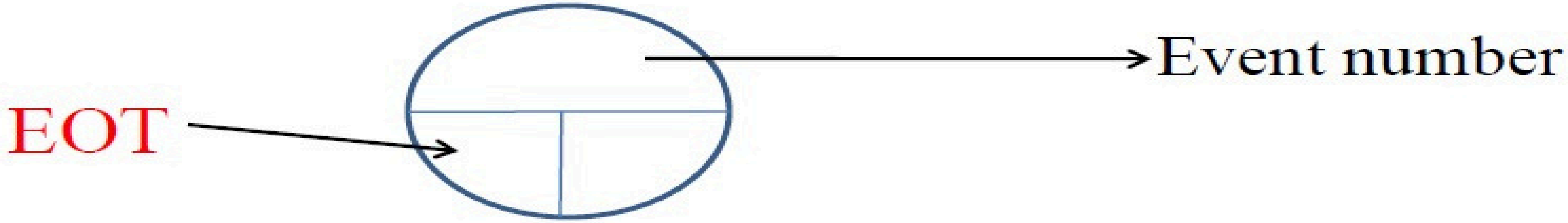
Occurrence time /Expected time for each event : As discussed earlier **PERT is an event oriented network** and here the importance is given to the occurrences of events . An event occurs at a point of time. The **Occurrence time /Expected time is classified as:**

(i) Earliest Expected Time of Occurrence of an event **(EOT).**

(ii) Latest Allowable Time of Occurrence an event **(LOT).**

STEP 5: ESTIMATE THE OCCURRENCE TIME FOR EACH EVENT AND COMPUTE THE SLACK

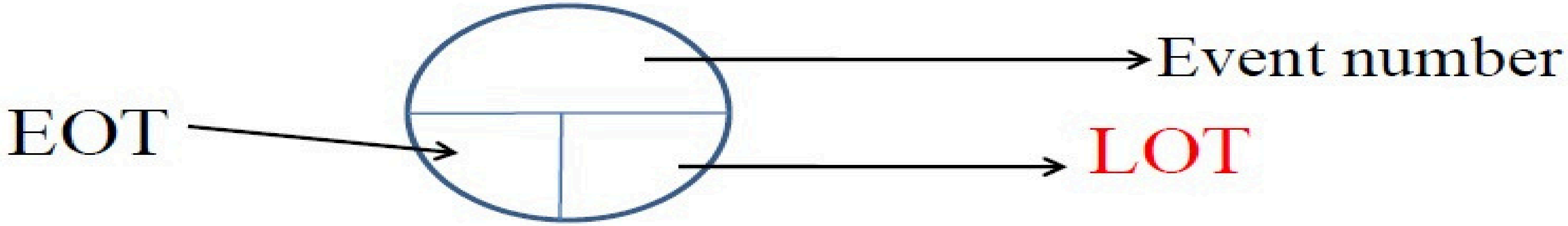
- The Earliest Expected Time of Occurrence (EOT) of an event is the earliest possible time of expecting that event to happen on the condition that all the preceding activities have been completed.
- The earliest expected time is the time when an event can expected to occur. It is generally illustrated in the left side of the bottom half of the circle representing the event [**left quadrant of the bottom half of the circle (EOT)**]. The upper half of the circle represents the **Event number**.



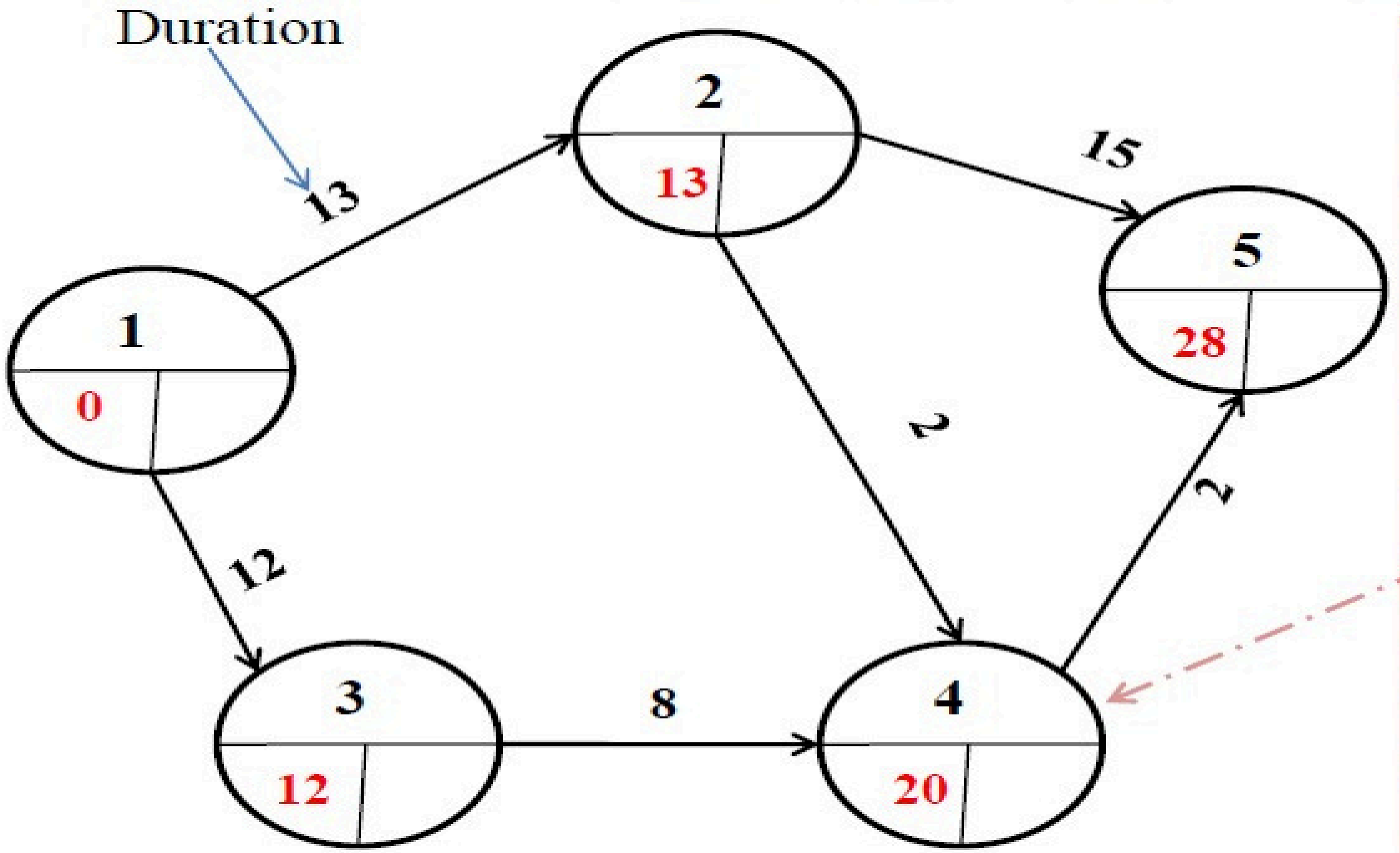
STEP 5: ESTIMATE THE OCCURRENCE TIME FOR EACH EVENT AND COMPUTE THE SLACK

The **Latest Allowable Time of Occurrence (LOT)** of an event is the latest possible time of expecting that event to happen without delaying the project beyond the stipulated time.

All projects are time bound ie. it has to be completed by some specific period of time. The latest time by which an event must occur to keep the project on schedule is called the latest allowable occurrence time. . It is generally illustrated in the right side of the bottom half of the circle representing the event [**right quadrant of the bottom half of the circle (EOT)**].



EARLIEST EXPECTED TIME OF OCCURRENCE (EOT)



An event occurs when all the activities leading to the event have been completed.

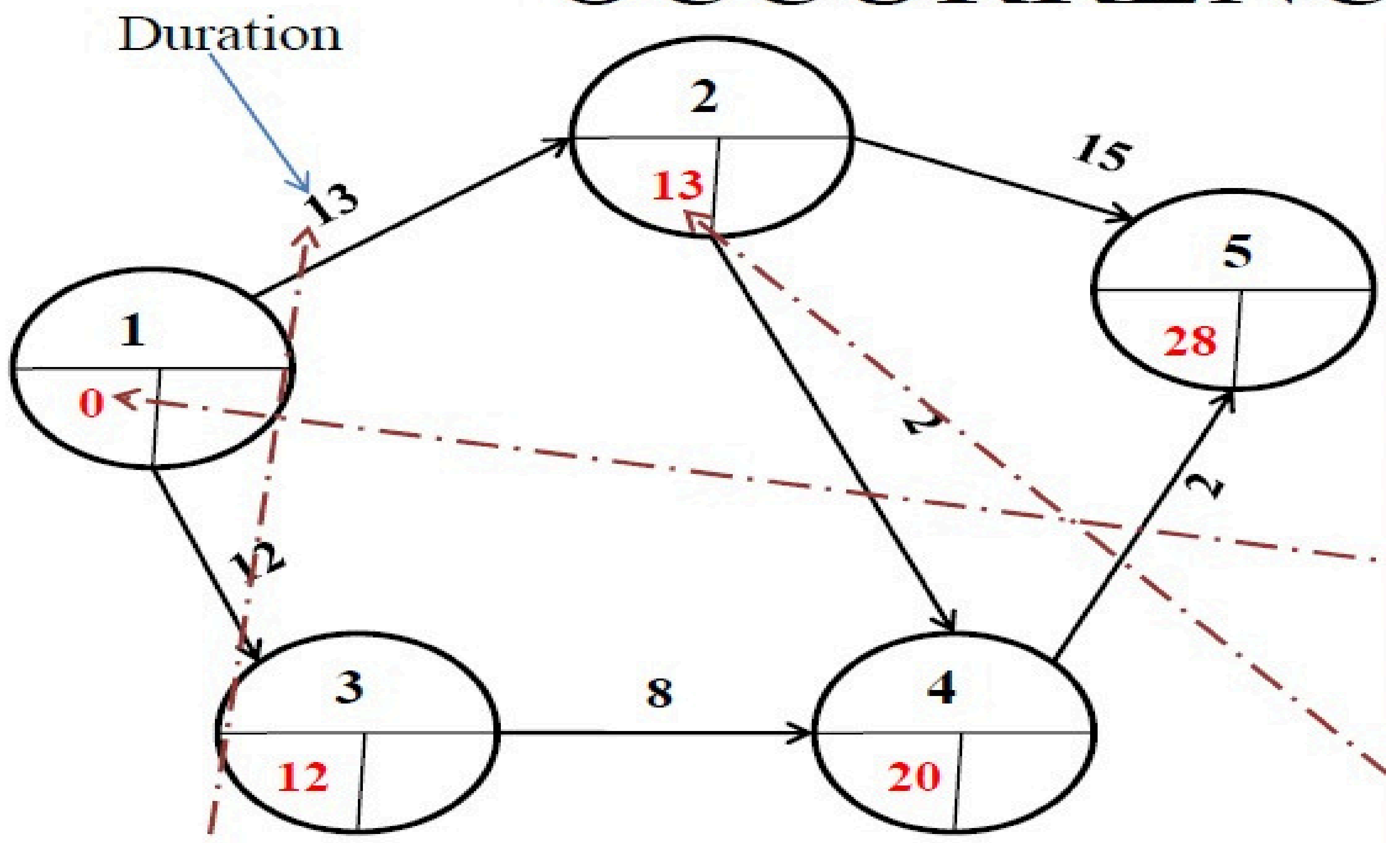
For example in the figure **event 4** occurs when the activity (2-4) and (3-4) are completed.

Activity (2-4) cannot begin unless 2 occurs, which in turn requires the completion of activity (1-2).

In other words the event 4 occurs when the activities **(1-2) (2-4) (1-3)** and **(3-4)** are completed or the event 4 occurs when the path **1-2-4 and 1-3-4** are completed.

The EOT of an event refers to the time when the event can be completed at the earliest . To obtain the EOT of various events we start from the first event and move forward to the last event. This computation procedure is called Forward Pass

EARLIEST EXPECTED TIME OF OCCURRENCE (EOT)



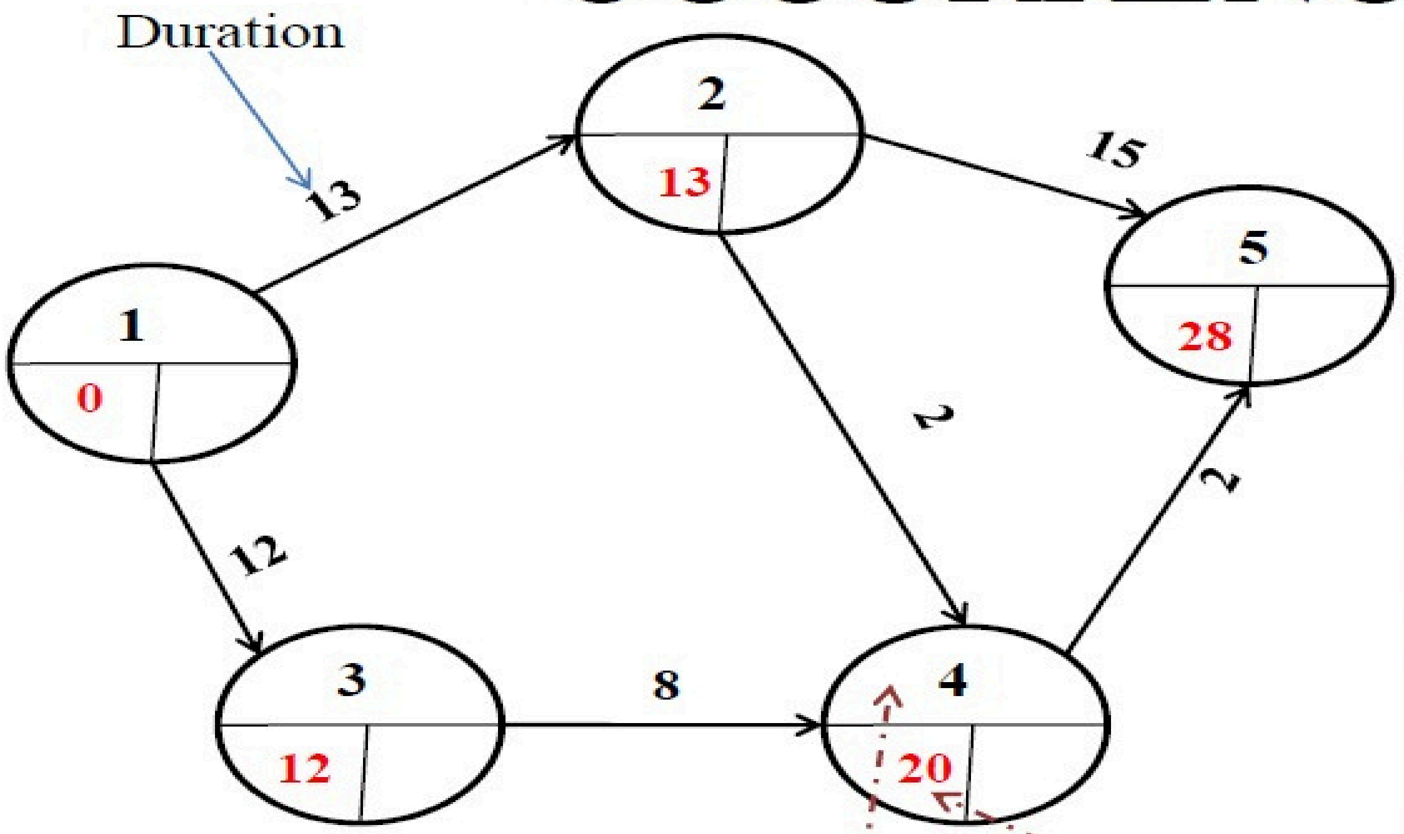
The EOT or the Earliest expected time is calculated by adding the expected times of all the activities along the path leading to the event.

Take the Earliest Expected Time (EOT) of **Occurrence of the Start Event as zero.**

For an event other than the start Event, find out all paths in the network which connect the Start node with the node representing the event under consideration and EOT (i) = Max [EOT(k)+ d (k,i)]

For Example, EOT of event 2 ie EOT (2) = Max [EOT(1) + 13] where d (k,i) represents the duration between event k and i in our case k= event 1 and i= event 2 d (i,j) =13, EOT of event 2= 13

EARLIEST EXPECTED TIME OF OCCURRENCE (EOT)



EOT (i) = Max [EOT(k)+ d (k,i)]
For an event under consideration, locate all the predecessor events and identify their earliest expected times.
With the earliest expected time of each event, add the time duration of the activity connecting that event to the event under consideration.
The maximum among all these values gives the Earliest Expected Time of Occurrence of the event.

For Example, EOT of event 4 Looking at event 4 we find paths leading to the event 4 ie 1-2-4 and 1-3-4 takes 15 weeks and 20 weeks respectively ie 13+2= 15 and 12+8=20, In general the EOT of an event is the duration of the longest path in this case it is 20.

EARLIEST EXPECTED TIME OF OCCURRENCE (EOT)

Hence the starting and finishing time for various activities obtained from this computation are **Earliest start time (EST)** and **Earliest finish time (EFT).**

Earliest start time (EST) is defined as the earliest possible time at which an activity can start. It is calculated by moving from first to last event in a network diagram.

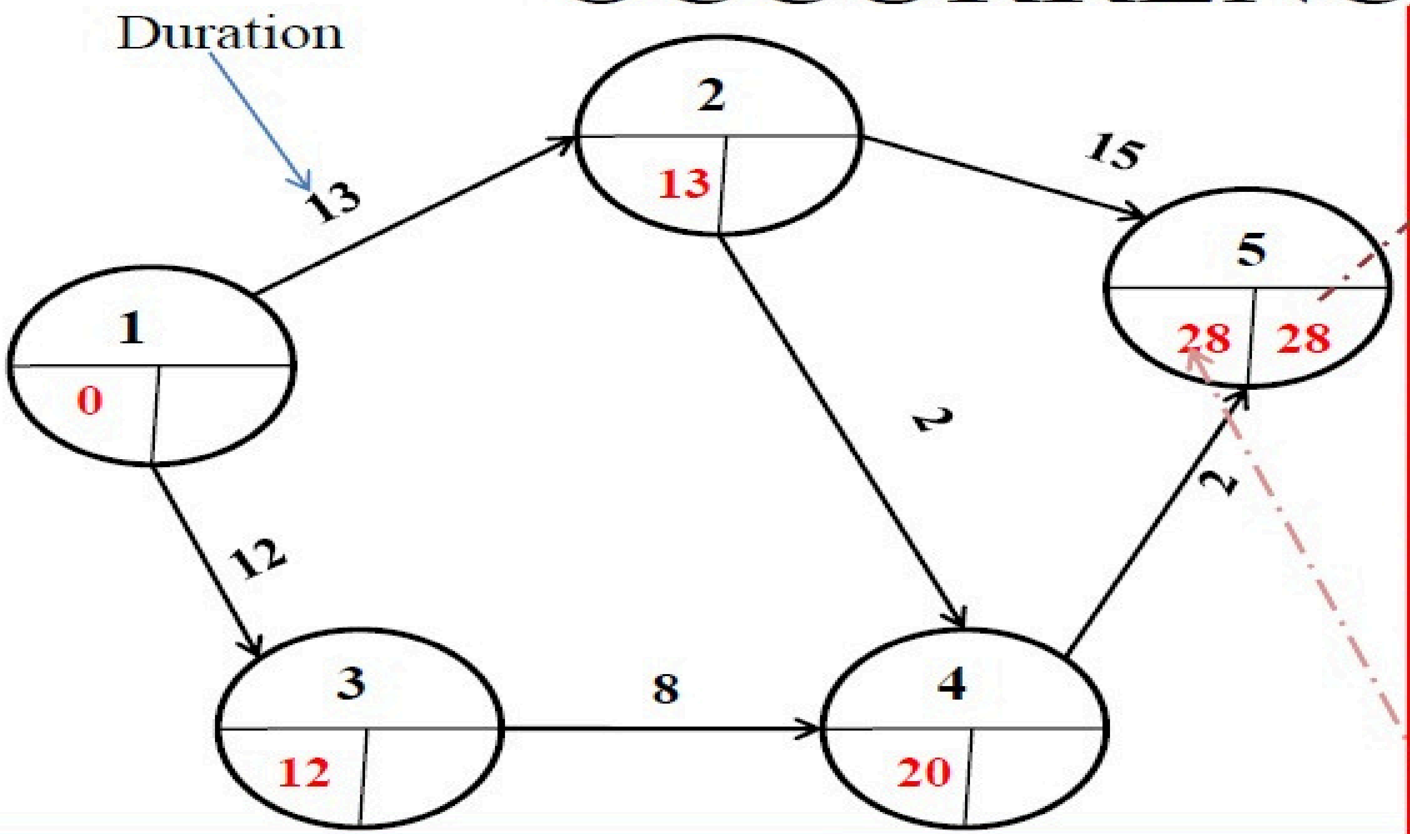
$$EST\ (i,j) = EOT\ (i)$$

Earliest Finish Time (EFT) of an activity is the earliest possible time of completing that activity. It is given by the formula.

$$EFT\ (i, j) = EOT\ (i) + d(i, j)$$

In other words, The Earliest Finish Time of an activity (EFT) = The Earliest Start Time of the activity (EST) + The estimated duration to carry out that activity [d(i,j)].

LATEST EXPECTED TIME OF OCCURRENCE (LOT)



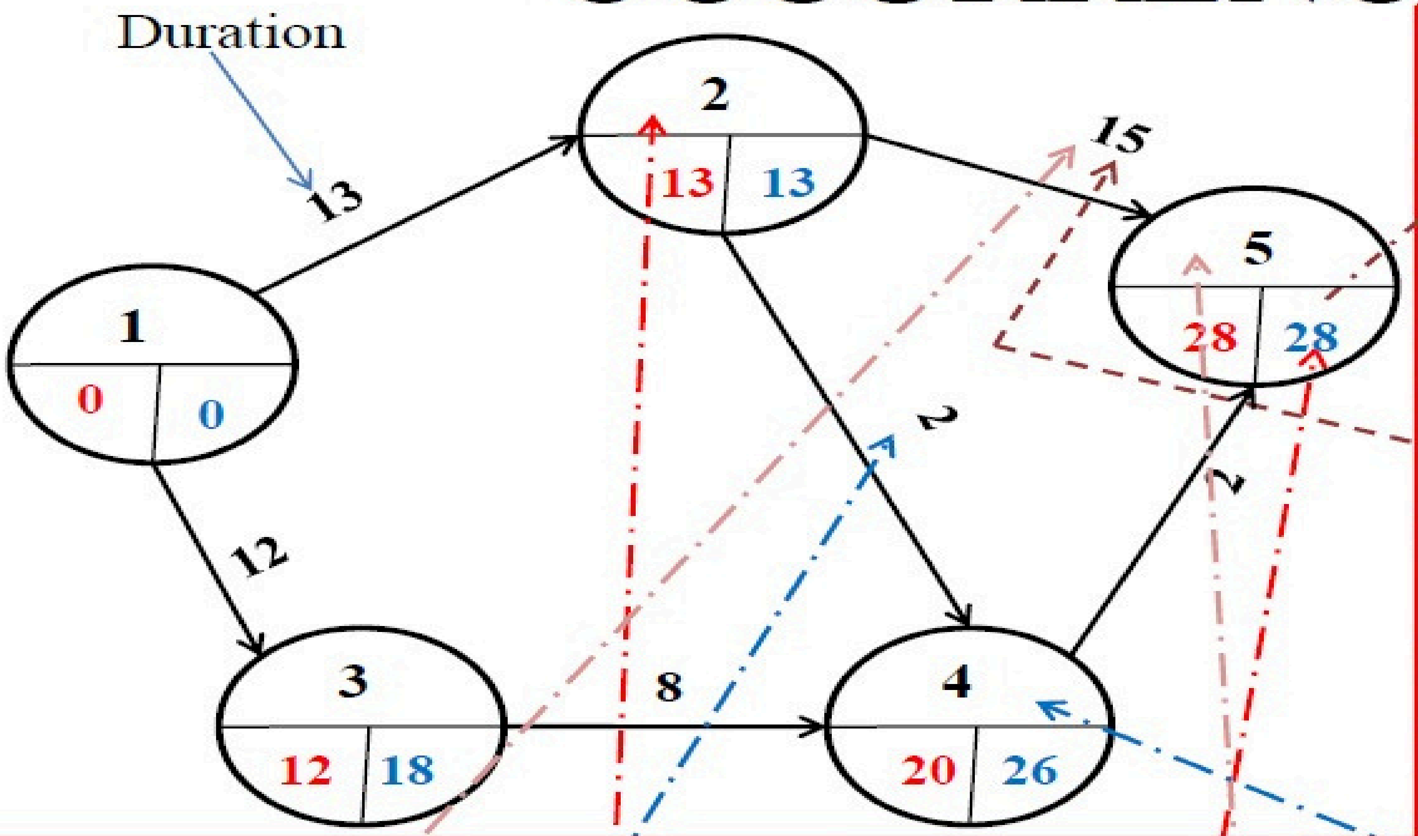
The **latest allowable time/ latest expected time of occurrence** is the allowable time by which an event can occur, given the time that is allowed for completion of the project.

The latest time by which an event must to keep the project on schedule is called the **latest allowable time/ latest expected time of occurrence** and is expressed as **(LOT)** .

Normally, the time allowed for the completion of the project is set equal to EOT of the end event. ie the project is supposed to be completed at the earliest possible time

At the end of the project EOT and LOT are set equal. To obtain the LOT of various events we start from the **LAST event** and move forward to the **FIRST event**. This computation procedure is called BACKWARD. The LOT of **Event 5 is equal to 28**

LATEST EXPECTED TIME OF OCCURRENCE (LOT)



The **latest allowable time/ latest expected time of occurrence(LOT) is calculated as follows**

LOT *(i)* = Min [LOT *(j)* – d *(i,j)*]

For example LOT of event 2 is equal to:

LOT (2)= Min[LOT (5)- 15]

d *(i,j) ie duration d(2,5) = 15*

LOT of event 2 = 28-15
= 13
LOT of event 2 = 26-2
= 24

Min of this is selected **ie 13**

LOT of event 4 = 28-2
= 26

Note: In the case of Event 2 which is connected to Events 5 and 4 so we have to consider the duration of activities (2-5) and (2-4) and the minimum LOT is selected using the above equation, this principle is applicable to all events

At the end of the project EOT and LOT are set equal. To obtain the LOT of various events we start from the **LAST event** and move forward to the **FIRST event**. This computation procedure is called BACKWARD. The LOT of **Event 5 is equal to 28**

LATEST EXPECTED TIME OF OCCURRENCE (EOT)

Hence the starting and finishing time for various activities obtained from this computation are **Earliest start time (EST)** and **Earliest finish time (EFT).**

Latest Finish time (LFT) of an activity is the latest possible time of completing that activity on the condition that all the other activities succeeding it are carried out as per the plan of the management and without delaying the project beyond the stipulated time.

$$\mathbf{LFT}\ (i, j) = \mathbf{LOT}\ (i)$$

Latest Start Time (LST) of an activity is the latest possible time of beginning that activity.. It is given by the formula.

$$\mathbf{LST}\ (i,j) = \mathbf{LFT}\ (i, j) - d\ (i, j)$$

In other words, The Latest Start Time of an activity (LST) = The Latest Finish Time of the activity (LFT) − The estimated duration to carry out that activity [d(i,j)].

Step 5: Estimate the occurrence time for each event and compute the slack

- Slack is denoted in Program Evaluation and Review Technique (PERT) as, how much an event can be delayed beyond its earliest start date, without causing any problems in the completion of the project by its due date
- **Slack of an Event**

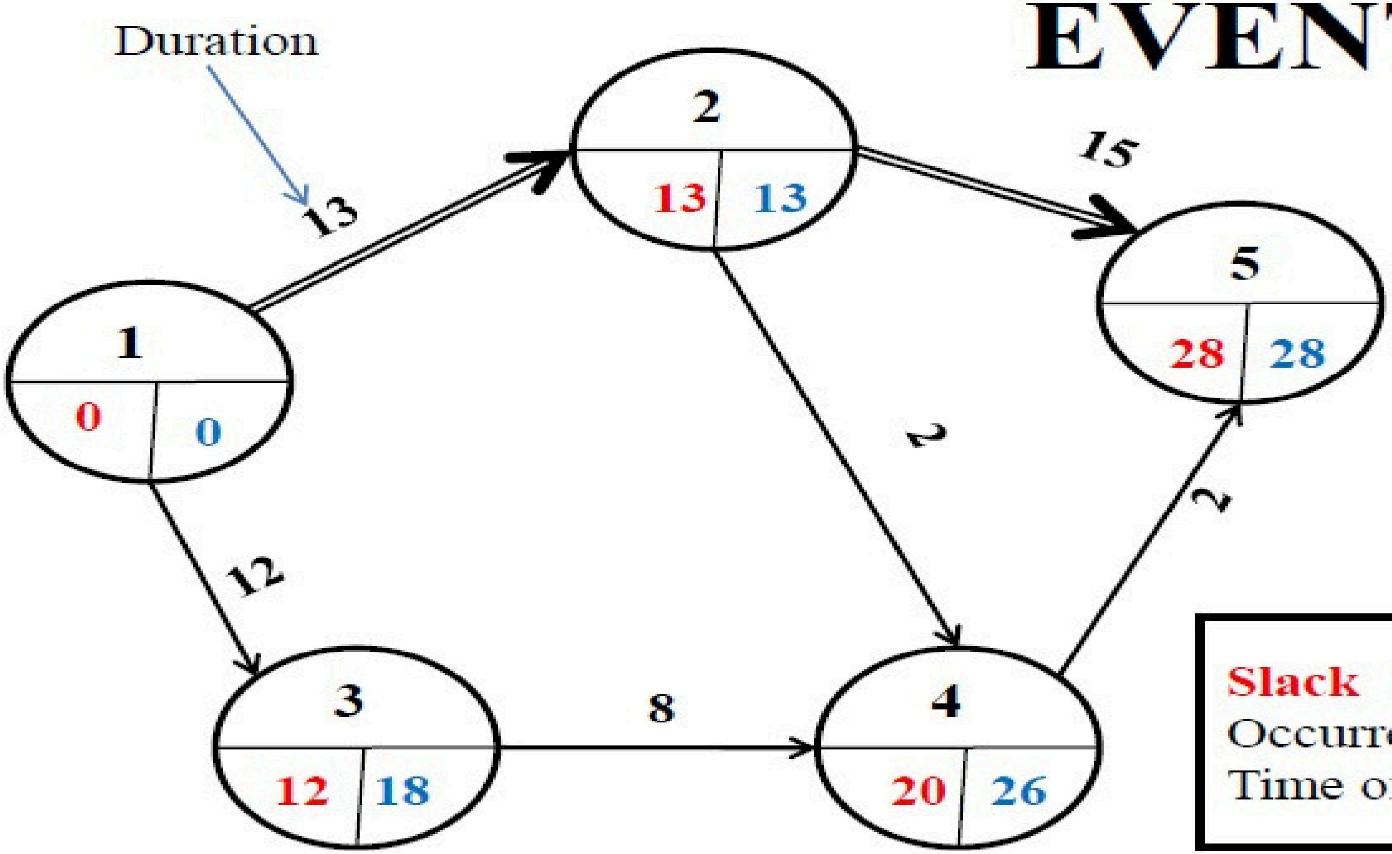
❖ The allowable time gap for the occurrence of an event is known as slack

❖ If there is a difference between Latest Allowable Time of Occurrence of the event (LOT) and Earliest Expected Time of Occurrence (EOT) , it indicate that the particular event can be delayed by that difference without changing the project duration.

❖ The slack of that event. It is given by the formula

❖ Slack of an event = Latest Allowable Time of Occurrence of the event (LOT) - Earliest Expected Time of Occurrence (EOT) of that event.

CALCULATION OF SLACK OF EVENTS



Events	LOT	EOT	SLACK
1	0	0	0
2	13	13	0
3	18	12	6
4	26	20	6
5	28	28	0

Slack of an event = Latest Allowable Time of Occurrence of the event **(LOT)** - Earliest Expected Time of Occurrence **(EOT)** of that event

Critical path: The critical path starts with the **beginning events**, terminates with the **end event** and is marked by events with **zero slack**. This is obviously the path on which there is no slack, no cushion. Other paths are slack path with some cushion. The critical path for the illustrative project is **(1-2-5)**. It is generally represented by double arrow

CRITICAL PATH

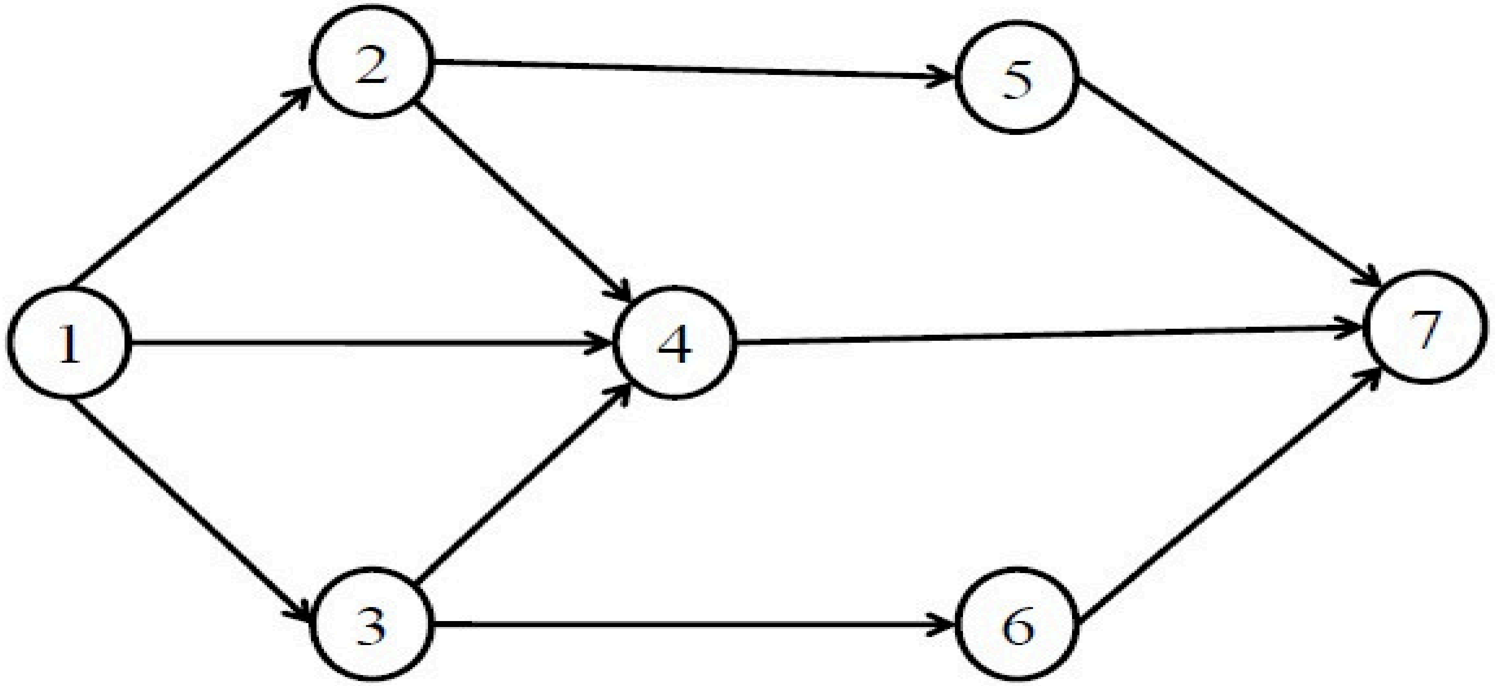
The critical path is the longest path of the network diagram. The activities in the critical path have an effect on the deadline of the project. If an activity of this path is delayed, the project will be delayed.

Critical path: The critical path is the longest path from the beginning event to the end event. Since the end can be reached ie., project completed, only when the longest path is traversed, the minimum time required for completing the project is the duration on the critical path. The duration on the critical path of the illustrative example **is 28 weeks ,It is the minimum time required to completing the project (It is already indicated by the EOT of event 5 the end event).**

Some Projects can have more than one critical path in a project, so that several paths run concurrently. This can be the result of multiple dependencies between tasks, or separate sequences that run for the same duration.

In Project management Critical Path is determined by identifying all paths of activities from the beginning of the network diagram until the end. After all alternative paths of activities are identified, the longest path that goes from the beginning until the end will be the critical path

PRACTICE PROBLEM



From the network given, find the expected time for each activity, Also find the critical path , Durations are given in days

Activity	t_o	t_m	t_p	Activity	t_o	t_m	t_p
1-2	1	2	3	3-4	2	4	6
1-3	2	3	4	3-6	3	5	7
1-4	3	6	8	4-7	14	18	20
2-4	4	6	8	5-7	4	6	8
2-5	4	5	7	6-7	5	7	9

SOLUTION

First construct a table as shown in the figure representing all the events and activities

Event	Predecessor event	Activity	to	tm	tp	te
1	0	--	--	--	--	--
2	1	1-2	1	2	3	2
3	1	1-3	2	3	4	3
4	1	1-4	3	6	8	5.63
	2	2-4	4	6	8	6
	3	3-4	2	4	6	4
5	2	2-5	4	5	7	5.16
6	3	3-6	3	5	7	5
7	4	4-7	14	18	20	17.66
	5	5-7	4	6	8	6
	6	6-7	5	7	9	7

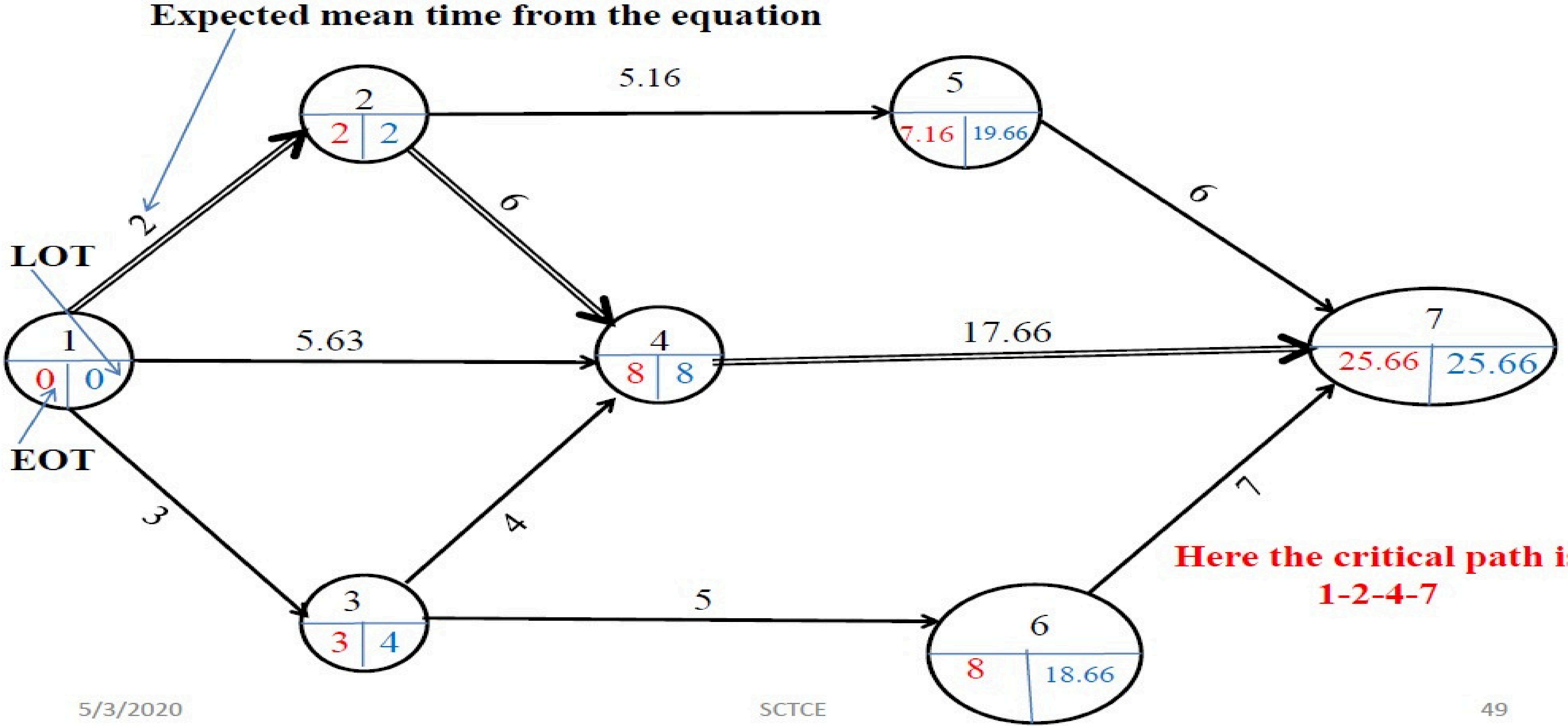
Find the expected mean time from the equation

$$t_e = \frac{t_o + 4\,t_m + t_p}{6}$$

Next find the LOT and EOT of all the events kindly refer next slide

PRACTICE PROBLEM

Answer:



Here the critical path is
1-2-4-7

PRACTICE PROBLEM

Event	Predecessor event	Activity	Duration in days				EOT	LOT	SLACK = LOT-EOT
			t_o	t_m	t_p	t_e			
1	**0**	--	--	--	--	--	**0**	**0**	**0**
2	**1**	**1-2**	1	2	3	2	**2**	**2**	**0**
3	1	1-3	2	3	4	3	3	4	1
4	1	1-4	3	6	8	5.63	**8**	**8**	**0**
	2	**2-4**	4	6	8	6			
	3	3-4	2	4	6	4			
5	2	2-5	4	5	7	5.16	7.16	19.66	12.5
6	3	3-6	3	5	7	5	8	18.66	10.00
7	**4**	**4-7**	14	18	20	17.66	**25.66**	**25.66**	**0**
	5	5-7	4	6	8	6			
	6	6-7	5	7	9	7			

Answer

Critical activities

Critical path is 1-2-4-7,Duration of the project is 25.66

Critical Path

- Tofind the critical path, need to determine the following quantities for eachactivity inthe network

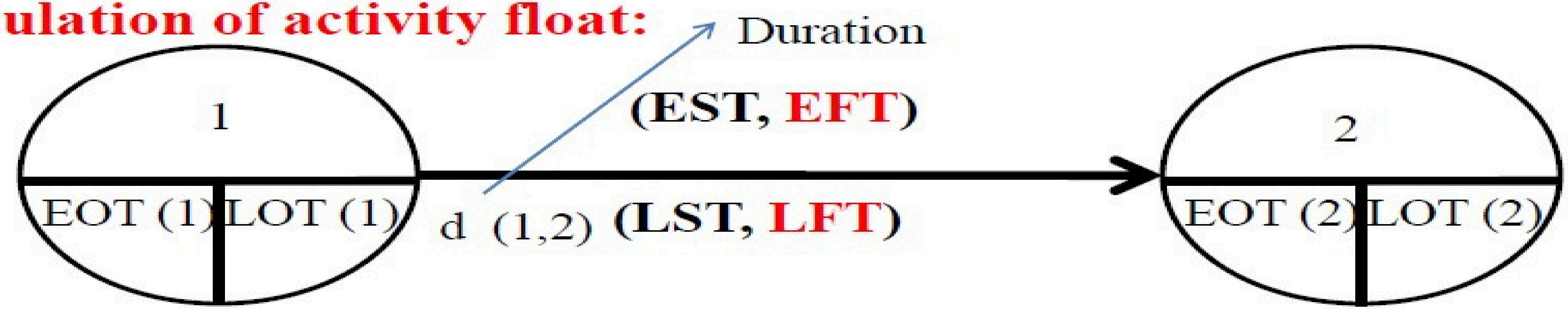
1. *Earliest start time* (*ES*): the earliest time an activity can begin without violation of immediate predecessor requirements
2. *Earliest finish time* (*EF*): the earliest time at which an activity can end
3. *Latest start time* (*LS*): the latest time an activity can begin without delaying the entire project
4. *Latest finish time* (*LF*): the latest time an activity can end without delaying the entire project

ACTIVITY FLOAT

- Float seeks to measure how much delay is acceptable. It sets up a control limit for delay. The total float of an activity is the time by which that activity can be delayed without delaying the whole project. It is given by the formula. The slack of an activity is the float of the activity
- Float, sometimes called slack, is the amount of time an activity, network path, or project can be delayed from the early start without changing the completion date of the project. Total float is the difference between the finish date of the last activity on the critical path and the project completion date
- There are four **types of floats**. **Total Float**, **Free Float**, **Interfering Float** and **Independent Float.**
- (i) **Total Float** : The **total float of** an activity represents the amount of time by which an activity can be delayed without delaying the project completion date.

ACTIVITY FLOAT

Calculation of activity float:



It is sometimes observed that certain activities have a difference between the maximum time available for completion and their actual duration . The difference is called **Float**

The **Total float (TF)** of an activity is the extra time available to complete the activity if it started early as possible, **without delaying** the completion of project . The **total float** of an activity is the time by which that activity can be delayed without delaying the whole project.

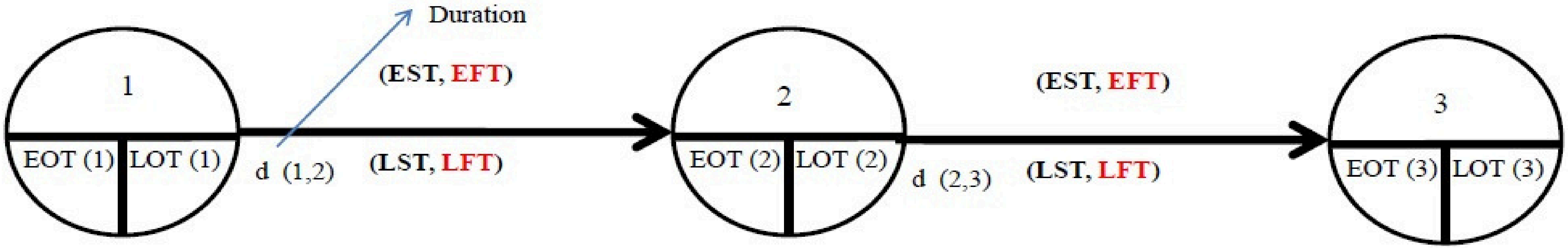
The **total float of an activity (1-2)= LOT (2)- EOT (1)- duration of activity** (d (1,2)

It is given by the formula Total Float of an Activity = Latest Finish Time (LFT) of the activity - Earliest Finish Time **(EFT)** of that activity

ACTIVITY FLOAT

- Since a delay in a critical activity will delay the execution of the whole project, the total float of a critical activity must be zero.
- Free float : Free Float is the amount of time that an activity can be delayed without delaying the early start date of any successor activity. Free float is a portion of the total float.
- Free float should be less than or equal to Total float.
- Free float is how long an activity can be delayed, without delaying the Early Start of its successor activity.
- **Free float** can be calculate the free float by subtracting the **Early Finish time of the activity** from the **Early Start date of the next activity.**

FREE FLOAT (FF) OF AN ACTIVITY

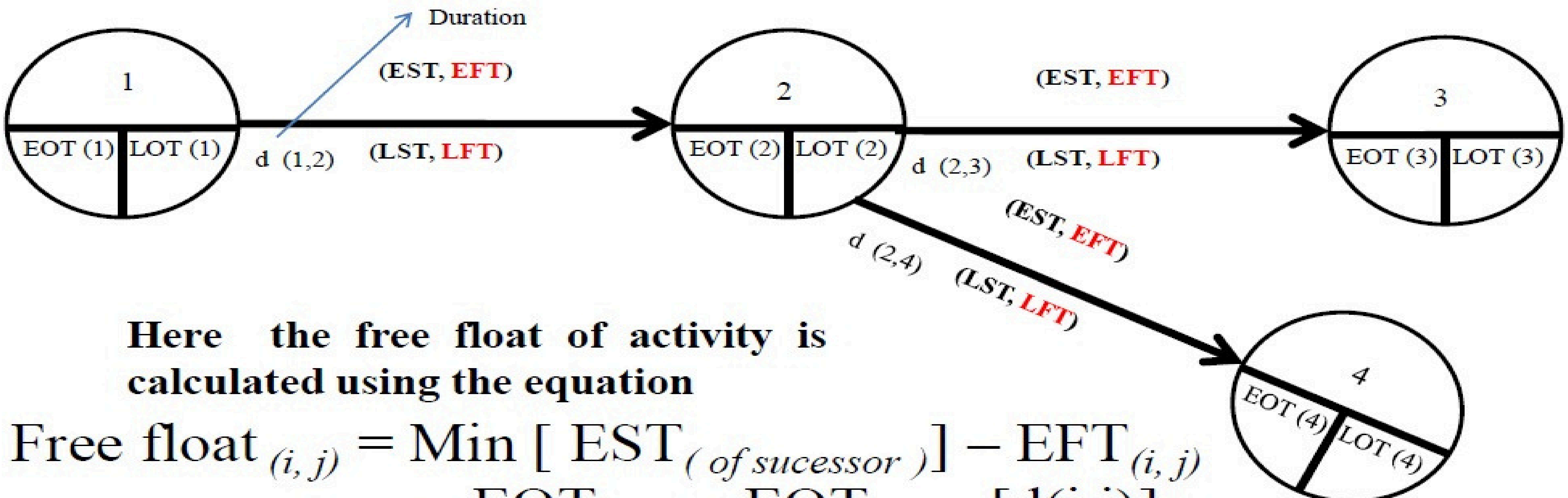


- Free $\text{float}_j = \text{EST}_{j+1} - \text{EFT}_j = \text{EET}_{head} - \text{EET}_{tail} - \text{D}$

$$\text{Free float}_{(1,2)} = \text{EST}_{(2,3)} - \text{EFT}_{(1,2)}$$

$$= \text{EOT}_{head} - \text{EOT}_{tail} - [d(i,j)]$$

FREE FLOAT OF AN ACTIVITY



Here the free float of activity is calculated using the equation

$$\text{Free float}_{(i,\,j)} = \text{Min}\,[\,\text{EST}_{(\,of\ sucessor\,)}] - \text{EFT}_{(i,\,j)}$$
$$= \text{EOT}_{head} - \text{EOT}_{tail} - [\text{d(i,j)}]$$

For example here for calculating the free float of activity (1-2) we have to take the minimum EST value of the successor activities (2-3) and (2-4)

INTERFERING FLOAT (IntF)

- **Interfering float :** Maximum amount by which an activity can be delayed without delaying the project but will cause delay to the Early Start of some following activity.

- It may be defined as: The time span in which the completion of an activity may occur and not delay the termination of the project but within which completion will delay the start of some other following activity.

- Interfering float activity (i-j) = late Finish time (LFT) of activity (i-j) – Smallest Early start (EST) of succeeding activity .

- The aggregate of free float and interfering float is equal to the total float.

- **Total float** activity (i-j) = **Free float** activity (i-j) + **interfering float** activity (i-j).

- This is equal to the **Head event slack** .

INDEPENDENT FLOAT (IF)

- The fourth float, **independent float**, is the amount of scheduling leeway (Amount of freedom) of an activity that is independent of the early starts and late finishes of any other activity.
- It may be formally defined as: The time span in which the completion of an activity may occur and not delay the termination of the project, not delay the start of any following activity, and not be delayed by any preceding activity.

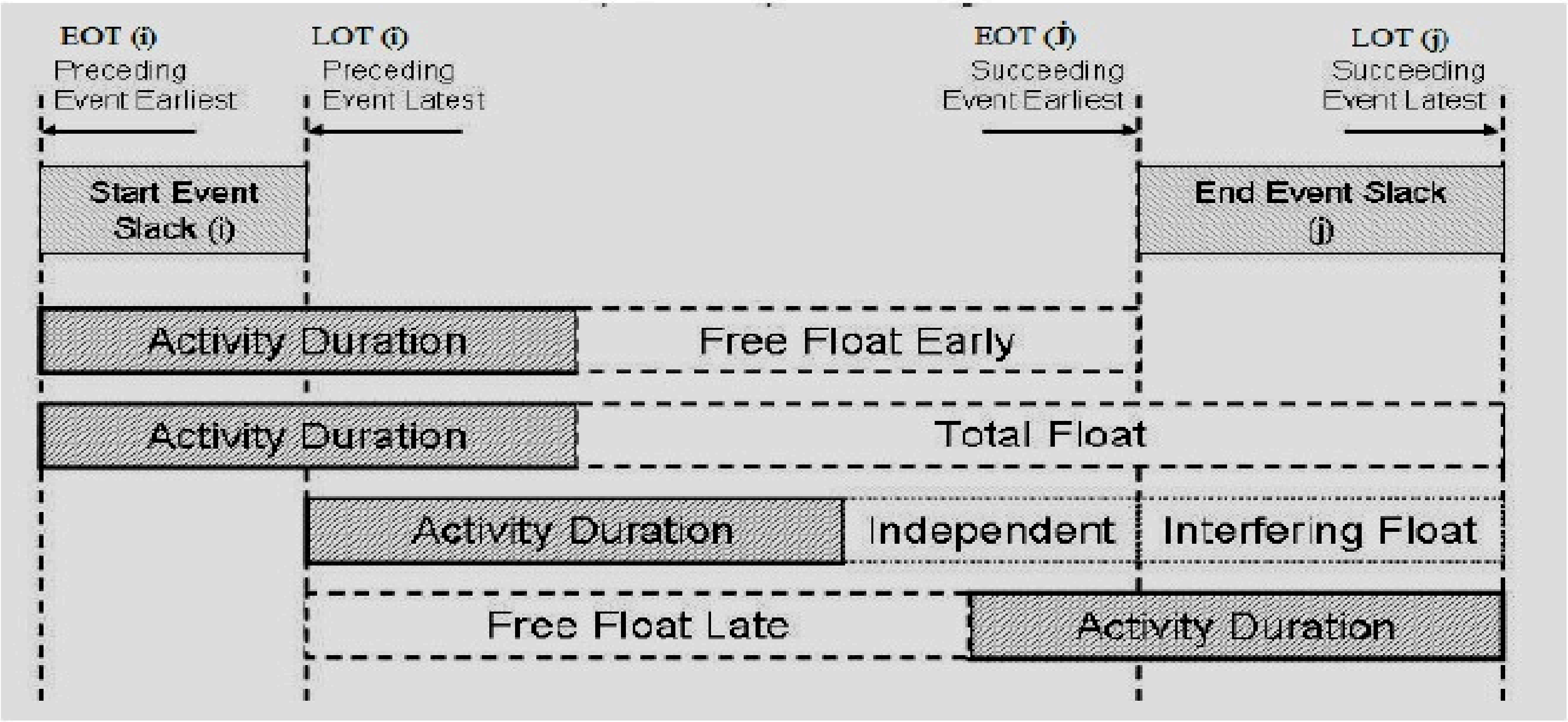
INDEPENDENT FLOAT (IF)

- The amount of scheduling flexibility available on the activity without displacing any other activity (before or after). It is the float available to the activity regardless of the timing of either node.
- This is calculated as

$$IF = EOT(j) - LOT(i) - d(ij)$$

- Therefore independent float is equal to free float minus tail event slack

ACTIVITY FLOAT



For an Activity {i-j}

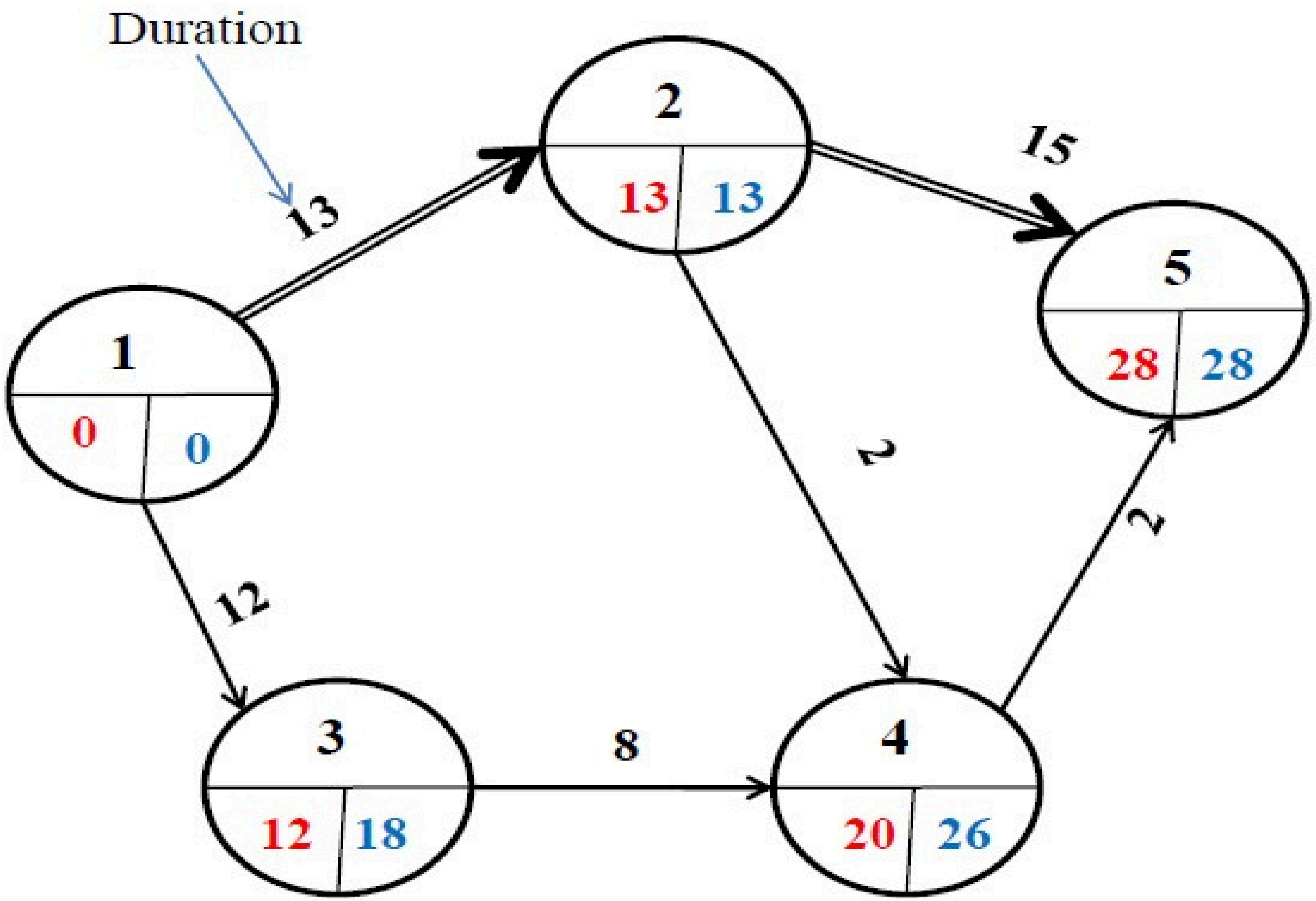
TF(i,j) = LOT(j) – EOT (i) – d (i,j)

FF (i,j) = EOT (j) – EOT (i) – d (i, j)

IF (i,j) = EOT(j) – LOT(i) – d(ij)

IntF (i,j) = TF (i,j)- FF (i,j)

PRACTICE PROBLEM



Calculate
EST
EFT
LST
LFT and all FLOATS

LST = LFT +d_ij
EST_ij = EOT_i
EFT_ij = EOT_i + d_ij
EFT = EST + d_ij

SOLUTION

LFT_ij = LOT_ j LST_ij = LFT_ij - d_ij TF_ij = LOT_j – EOT_i – d_ij FF_ij = EOT_j – EOT_i – d_ij

Activity (i,j)	Duration	Tail event		Head event		Activity time (in weeks)				FLOAT in week			
		EOT (i)	LOT (i)	EOT (j)	LOT (j)	EST	EFT	LST	LFT	TL	FF	IF	Int F
1-2	13	0	0	13	13	0	13	0	13	0	0	0	0
1-3	12	0	0	12	18	0	12	6	18	6	0	0	6
2-4	2	13	13	20	26	13	15	24	26	11	5	5	6
3-4	8	12	18	20	26	12	20	18	26	6	0	6	6
2-5	15	13	13	28	28	13	28	13	28	0	0	0	0
4-5	2	20	26	28	28	20	22	26	28	6	6	0	0

IF_ij = EOT_j – LOT_i – d_ij
IntF = TF_ij – FF_ij

Total float of critical path =0

Assignment question 4

A project consists of 12 activities and their time estimates are shown below

Activity	Time		
	t_o	t_m	t_p
1-2	1	1.5	5
2-3	1	2	3
2-4	1	3	5
3-5	3	4	5
4-5	2	3	4
4-6	3	5	7
5-7	4	5	6
6-7	6	7	8
7-8	2	4	6
7-9	5	6	8
8-10	1	2	3
9-10	3	5	7

Draw the network diagram b) Determine the critical path c) Calculate event slack and activity floats (d) Find the standard deviation of the critical path duration

Assignment question 5

For a small project of 12 activities, the details are given below. Draw the network and find earliest occurrence time, critical path and project completion time.

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Dependence				B,C	A	C	E	E	D,F,H	E	I,J	G
Duration(days)	9	4	7	8	7	5	10	8	6	9	10	2

What are the steps involved in PERT analysis. Explain event slack

Difference between PERT and CPM

	PERT	CPM
Meaning	PERT is a project management technique, used to manage uncertain activities of a project.	CPM is a statistical technique of project management that manages well defined activities of a project.
What is it?	A technique of planning and control of time.	A method to control cost and time.
Orientation	Event-oriented	Activity-oriented
Evolution	Evolved as Research & Development project	Evolved as Construction project
Model	Probabilistic Model	Deterministic Model
Focuses on	Time	Time-cost trade-off
Estimates	Three time estimates	One time estimate
Appropriate for	High precision time estimate	Reasonable time estimate
Management of	Unpredictable Activities	Predictable activities
Nature of jobs	Non-repetitive nature	Repetitive nature
Critical and Non-critical activities	No differentiation	Differentiated
Suitable for	Research and Development Project	Non-research projects like civil construction, ship building etc.
Crashing concept	Not Applicable	Applicable

Advantages of PERT/CPM

- Especially useful when scheduling and controlling large projects.
- Straightforward concept and not mathematically complex.
- Graphical networks aid perception of relationships among project activities.
- Critical path & slack time analyses help pinpoint activities that need to be closely watched.
- Project documentation and graphics point out who is responsible for various activities.
- Applicable to a wide variety of projects.
- Useful in monitoring schedules and costs.

Limitationsof PERT/CPM

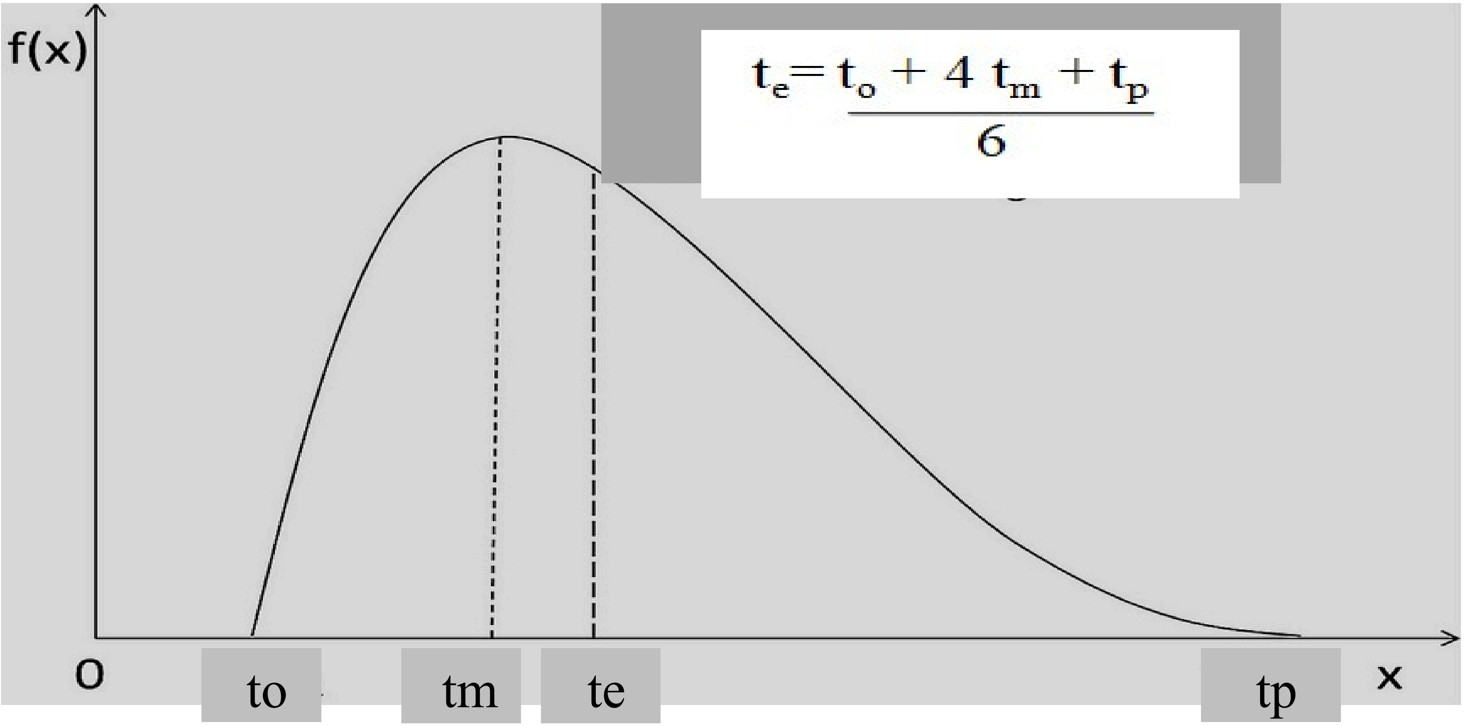
- Assumes clearly defined, independent, & stable activities
- Specified precedence relationships
- Activity times (PERT) follow beta distribution
- Subjective time estimates
- Over-emphasis on critical path

Probability of completion time of a project

- Itwill be necessaryto knowtheprobabilityof completing a project within the scheduled completion time once the critical path is identified .
- Given the uncertainties in accurately estimating activity durations, it isalso necessary to know what is the probability of completing the project by the deadline.

TIME ESTIMATE /DURATION OF PERT FOLLOWS BETA DISTRIBUTION

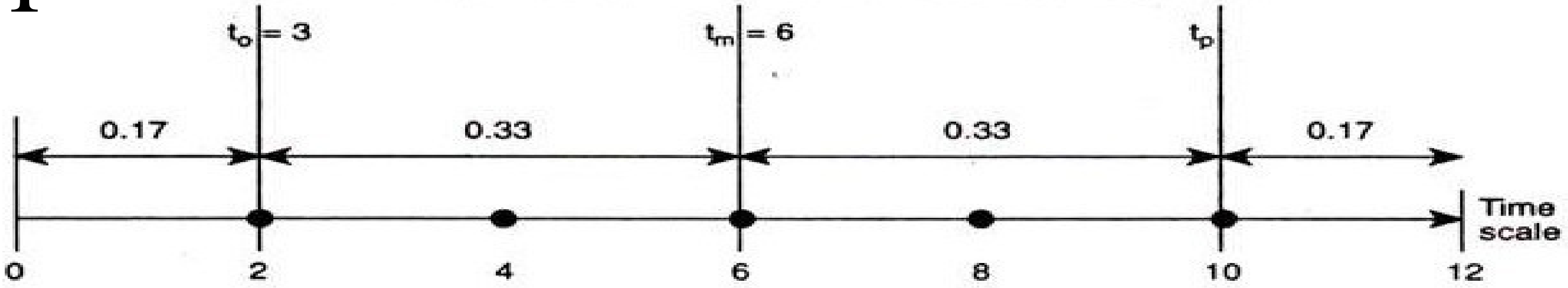
Beta-distribution



The general shape of the Beta-distribution is shown in the figure. It is seen that the peak of the curve or the mode corresponds to the most likely time tm. This peak may take up any position within the range of distribution to conform to the characteristics of the activity under consideration.

The range of the distribution roughly determines /define the optimistic and pessimistic times .Because these time estimates represents the extreme cases having little chance of occurrences therefore having very little probability. In Beta distribution a simple approximation can be made for obtaining the activity mean time

Example for Beta distribution



When the probability follows beta distribution (as assumed in PERT), and in the scale of time, time units 12 represents 100 per cent probability, then time units 6 is ~~0.5 or 50 per cent probability.~~ The most likely estimate (tm) is a probability of 0.5. As we have noted in the averaging formula the weightage for to tm and tp are 1, 4, and 1, respectively.

$$t_e = \frac{t_o + 4\,t_m + t_p}{6}$$

Note :The probability of occurrence of pessimistic (tp) and optimistic (to) time has less chances of attainment.

ASSUMPTIONS For calculating expected mean time and variance

- Assume thatthe mean critical path will turnout tobe the longest path through the project network. This is only a rough approximation, since the assumption occasionally does not hold in the usual case where some of the activity durations do not equal their means. Fortunately, when the assumption does not hold, the true longest path commonly is not much longer than the mean critical path. This approximation will enable us to calculate expected mean time

Assume that the durations of the activities on the mean critical path are statistically independent. This assumption should hold if the activities are performed truly independently of each other. However, the assumption becomes only a rough approximation if the circumstances that cause the duration of one activity to deviate from its mean also tend to cause similar deviations for some other activities. This approximation will enable us to obtain the variance.

PERT ASSUMPTIONS

- Assumption
  - –Acritical path can be determined by using the mean completion times for the activities.
  - –The project mean completion time is determined solely by the completion time of the activities on the critical path.

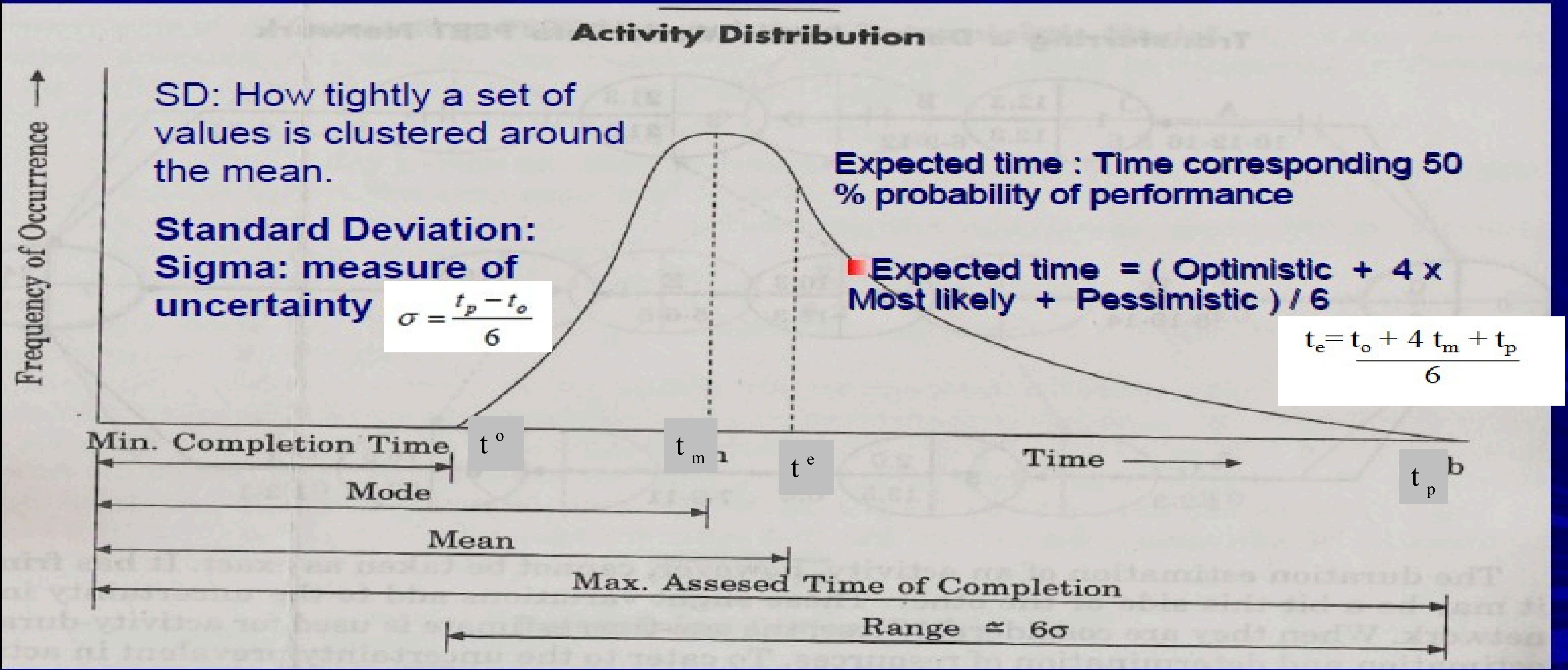
Assumption

- Thereareenough activities on the critical path so that the distribution of the overall project completion time can be approximated by the normal distribution.

Assumption

–Thetime tocomplete one activity is independent of the completion time of any other activity.

Expected Time & Standard Deviation: Beta Distribution



TIME ESTIMATE /DURATION

The expected mean time is obtained from the equation

$$t_e = \frac{t_o + 4\,t_m + t_p}{6}$$

The 3 estimates of time aresuch that

$$t_o \leq t_m \leq t_p$$

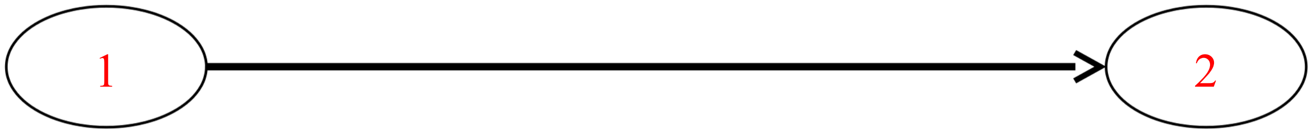
Therefore the range for the time estimate is $t_p - t_o$

The time taken by an activity in a project network follows a distribution with a standard deviation of one sixth of the range, approximately.

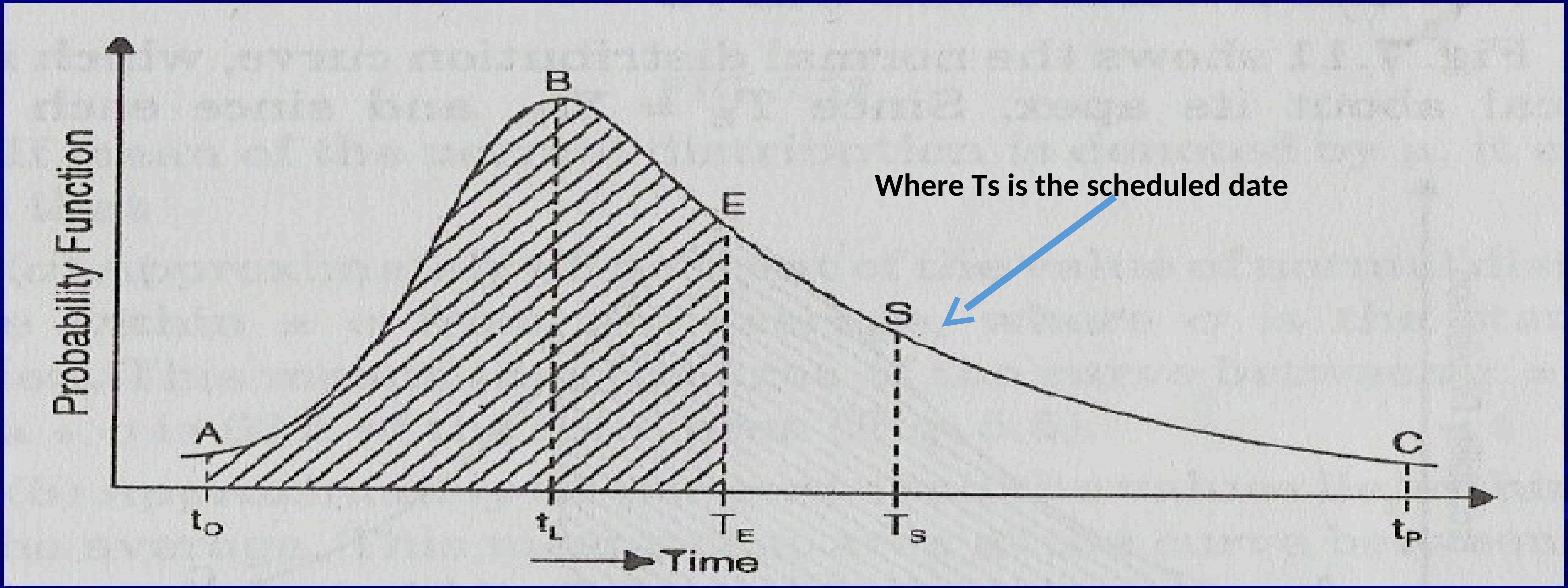
i.e., The standard deviation = $\sigma = \dfrac{t_p - t_o}{6}$

and the variance = $\sigma^2 = \left(\dfrac{t_p - t_o}{6}\right)^2$

Variability in PERT analysisis measured by variance or its square root, standard deviation. For determining the standard deviation of the duration we require the probability distribution of the activity distribution. We have only three values from the distribution ie t_o, t_m, t_p.



Probability of Meeting The Schedule Date



$$\text{Probability} = \frac{\text{area under } ABS}{\text{area under } ABC}$$

TIME ESTIMATE in PERT

Probability for Project Duration

The probability of completing the project within the scheduled time (Ts) or contracted time may be obtained by using the standard normal deviate where Te is the expected time of project completion.

$$Z_0 = \frac{T_s - T_e}{\sqrt{\Sigma\sigma^2 \text{ in critical path}}}$$

Normal deviate

Probability of completing the project within the scheduled time is,

P(T≤ Ts)=P (Z≤Z0) (Z, standard normal deviate from normaltables)

Note: though individual activities assume random (bet distribution) but Te of the project as a whole assume normal distribution

Normal Deviate (x): Distance from the mean expressed in terms of sigma

1. Normal Deviate = 0, it is the expected time, probability of completion = 50 %

2. Normal Deviate = 1, probability of completion = 84 %.

3. Normal Deviate = -1, probability of completion = 16 %

Normal Distribution Function

Normal deviate (+)	*Probability* (%)	*Normal deviate* (+)	*Probability* (%)
0	50.0	0	50.0
−0.1	46.0	+0.1	54.0
−0.2	42.1	+0.2	57.9
−0.3	38.2	+0.3	51.8
−0.4	34.5	+0.4	65:5
−0.5	30.8	+0.5	69.2
−0.6	27.4	+0.6	72.6
−0.7	24.2	+0.7	75.8
−0.8	21.2	+0.8	78.8
−0.9	18.4	+0.9	81.6
−1.0	15.9	+1.0	84.1
−1.1	13.6	+1.1	86.4
−1.2	11.5	+1.2	88.5
−1.3	9.7	+1.3	90.3
−1.4	8.1	+1.4	91.9
−1.5	6.7	+1.5	93.3
−1.6	5.5	+1.6	94.5
−1.7	4.5	+1.7	95.5
−1.8	3.6	+1.8	96.4
−1.9	2.9	+1.9	97.1
−2.0	2.3	+2.0	97.7
−2.1	1.8	+2.1	98.2
−2.2	1.4	+2.2	98.6
−2.3	1.1	+2.3	98.9
−2.4	0.8	+2.4	99.2
−2.5	0.6	+2.5	99.4
−2.6	0.5	+2.6	99.5
−2.7	0.3	+2.7	99.7
−2.8	0.3	+2.8	99.7
−2·9	0.2	+2.9	99.8
−3.0	0.1	+3.0	99.9

TIME ESTIMATE in PERT

The standard deviation of an activity

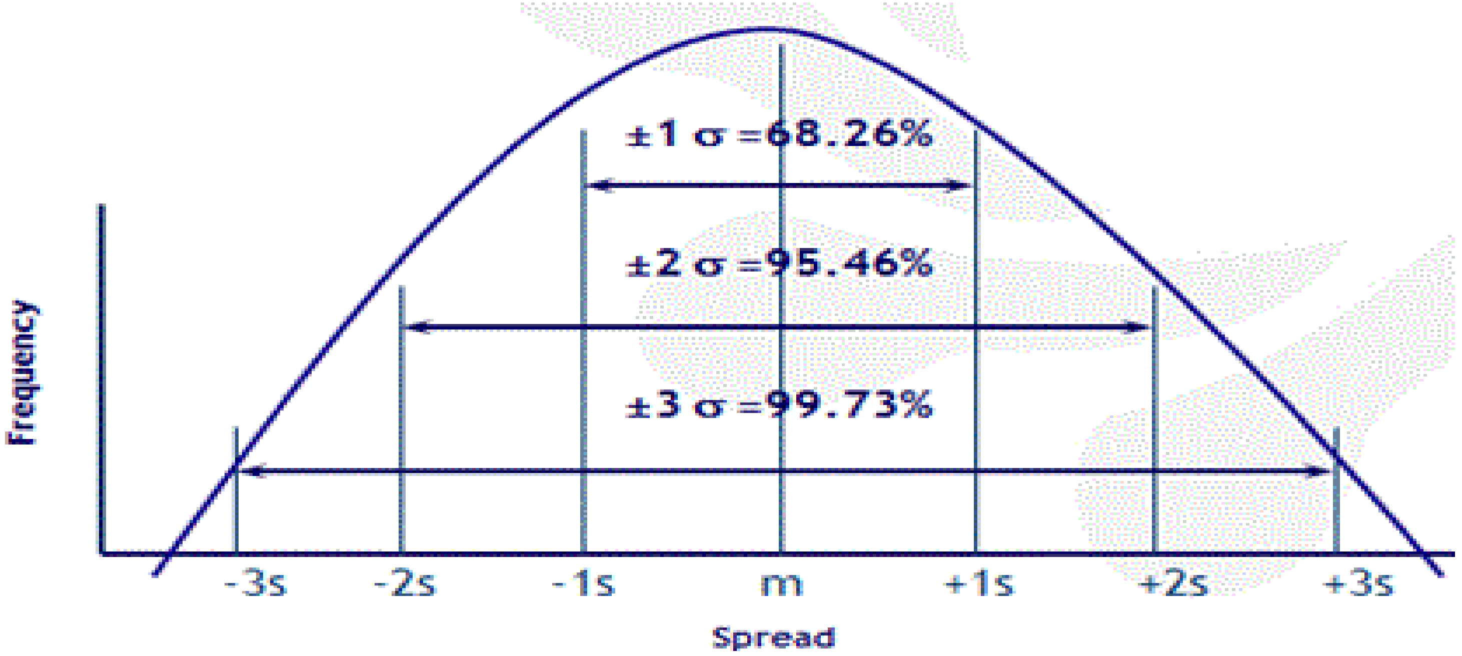
$$\sigma = \frac{t_p - t_o}{6}$$

Degree of variationfromthe average-(mean)

Indicates the standard error in the estimateandprovidesanidea ofits accuracy

the larger the standard deviation (spreadbetweentheoptimistic and pessimistic estimates), the larger the risk in the estimate.

± 1 standard deviation = 68.26% ± 2 standard deviations = 95.46% ± 3 standard deviations = 99.73% ± 6 standard deviations = 99.999998% - also known as Six Sigma



PRACTICE PROBLEM

Find the probabilityofcompletingtheprojectin24days,20days and 26 days

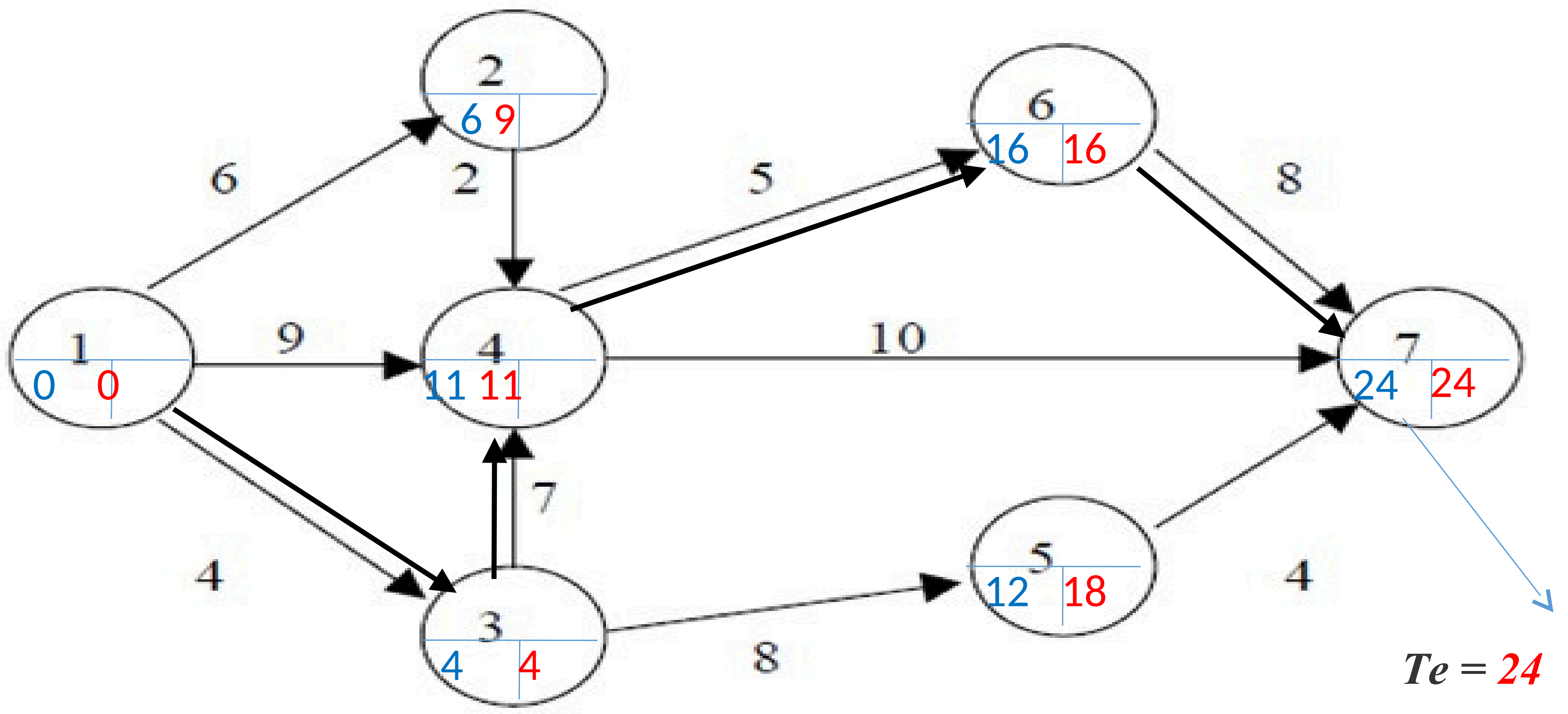
Activity i j	Activity Name	T_0	t_m (in days)	t_p
1-2	A	4	6	8
1-3	B	2	3	10
1-4	C	6	8	16
2-4	D	1	2	3
3-4	E	6	7	8
3-5	F	6	7	14
4-6	G	3	5	7
4-7	H	4	11	12
5-7	I	2	4	6
6-7	J	2	9	10

$$t_e = \frac{t_o + 4\,t_m + t_p}{6}$$

Solution: *Fist find the* expected mean time

Activity	t_o	t_m	t_p	t_e
1-2	4	6	8	6
1-3	**2**	**3**	**10**	**4**
1-4	6	8	16	9
2-4	1	2	3	2
3-4	**6**	**7**	**8**	**7**
3-5	6	7	14	8
4-6	3	5	7	5
4-7	4	11	12	10
5-7	2	4	6	4
6-7	**2**	**9**	**10**	**8**

SOLUTION :*find EOT and LOT and obtain the critical path*



CRITICAL PATH: 1-3-4-6-7

SOLUTION: Find the standard deviation (σ) and variance (σ^2)

$$t_e = \frac{t_o + 4\,t_m + t_p}{6}$$

Activity	t_o	t_m	t_p	t_e	$\sigma = \frac{t_p - t_o}{6}$	σ^2
1-2	4	6	8	6	0.666667	0.444444
1-3	**2**	**3**	**10**	**4**	**1.333333**	**1.777778**
1-4	6	8	16	9	1.666667	2.777778
2-4	1	2	3	2	0.333333	0.111111
3-4	**6**	**7**	**8**	**7**	**0.333333**	**0.111111**
3-5	6	7	14	8	1.333333	1.777778
4-6	3	5	7	5	0.666667	0.444444
4-7	4	11	12	10	1.333333	1.777778
5-7	2	4	6	4	0.666667	0.444444
6-7	**2**	**9**	**10**	**8**	**1.333333**	**1.777778**

Calculation of normal deviate

Ts= The scheduled completion time

Te = The earliest completion time

Zo= The probability factor/ Normal deviate

$$Z_0 = \frac{T_s - T_e}{\sqrt{\Sigma\sigma^2} \text{ in critical path}}$$

→ **1.777778+ 0.111111+ 0.444444+ 1.777778 = 4.12**

*Te = **24***

Calculation of normal deviate

- Let us find theprobabilityofcompletingtheproject in 24 days, 20 days and 26 days

Here

$$Z_0 = \frac{T_s - T_e}{\sqrt{\Sigma\sigma^2 \text{ in critical path}}}$$

Normal deviate

And Ts for case 1 is 24 days, case 2 it is 20 days and case 3 it is 26 days

For finding the probabilities we have to find Zo for all the cases

Case 1 $= \frac{24-24}{4.12} = 0$

Case 2 $= \frac{20-24}{4.12} = -0.49$

Case 2 $= \frac{26-24}{4.12} = 0.49$

Normal Deviate (x): Distance from the mean expressed in terms of sigma

1. Normal Deviate = 0, it is the expected time, probability of completion = 50 %

2. Normal Deviate = 1, probability of completion = 84 %.

3. Normal Deviate = -1, probability of completion = 16 %

Normal Distribution Function

Zo

Normal deviate (+)	*Probability* (%)	*Normal deviate* (+)	*Probability* (%)
0	50.0	0	50.0
−0.1	46.0	+0.1	54.0
−0.2	42.1	+0.2	57.9
−0.3	38.2	+0.3	51.8
−0.4	34.5	+0.4	65:5
−0.5	30.8	+0.5	69.2
−0.6	27.4	+0.6	72.6
−0.7	24.2	+0.7	75.8
−0.8	21.2	+0.8	78.8
−0.9	18.4	+0.9	81.6
−1.0	15.9	+1.0	84.1
−1.1	13.6	+1.1	86.4
−1.2	11.5	+1.2	88.5
−1.3	9.7	+1.3	90.3
−1.4	8.1	+1.4	91.9
−1.5	6.7	+1.5	93.3
−1.6	5.5	+1.6	94.5
−1.7	4.5	+1.7	95.5
−1.8	3.6	+1.8	96.4
−1.9	2.9	+1.9	97.1
−2.0	2.3	+2.0	97.7
−2.1	1.8	+2.1	98.2
−2.2	1.4	+2.2	98.6
−2.3	1.1	+2.3	98.9
−2.4	0.8	+2.4	99.2
−2.5	0.6	+2.5	99.4
−2.6	0.5	+2.6	99.5
−2.7	0.3	+2.7	99.7
−2.8	0.3	+2.8	99.7
−2·9	0.2	+2.9	99.8
−3.0	0.1	+3.0	99.9

Final answer

- From the probability table
- The probabilityofcompleting the project for Case 1= 50 %
- Case 2= 30.8%
- Case 3= 69.2%

Project Crashing

- Projects will sometimes have deadlines that are impossible to meet using normal procedures
- By using exceptional methods it may be possible to finish the project in less time than normally required
- However, this usually increases the cost of the project
- Reducing a project’s completion time is called *crashing*

Project Crashing

- Crashing a project startswith usingthe*normal time* to create the critical path
- The *normal cost* is the cost for completing the activity using normal procedures
- If the project will not meet the required deadline, extraordinary measures must be taken
- The *crash time* is the shortest possible activity time and will require additional resources
- The *crash cost* is the price of completing the activity in the earlier-than-normal time

Factors to Consider when Crashing

- The amount by which an activity is crashed is, in fact, permissible.
- Taken together, the shortened activity durations will enable one to finish the project by the due date.
  The total cost of crashing is as small as possible.
-

Steps in Project Crashing

- Compute the crash cost per time period. For crash costs assumed linear over time:

$$\text{Crash cost per period} = \frac{(\text{Crash cost} - \text{Normal cost})}{(\text{Normal time} - \text{Crash time})}$$

- Using current activity times, find the critical path
- If there is only one critical path, then select the activity on this critical path that (a) can still be crashed, and (b) has the smallest crash cost per period. Note that a single activity may be common to more than one critical path
- Update all activity times.

Four Steps to Project Crashing

1. Findthe normal critical path and identifythecriticalactivities
2. Compute thecrashcost per week (orothertimeperiod)for all activities in the network using the formula

$$\text{Crash cost/Time period} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}$$

Four Steps to Project Crashing

3. Select the activity on the critical path with the smallest crash cost per week and crash this activity to the maximum extent possible or to the point at which your desired deadline has been reached
4. Check to be sure that the critical path you were crashing is still critical. If the critical path is still the longest path through the network, return to step 3. If not, find the new critical path and return to step 2.

“Linear programming is another approach to finding the best project crashing schedule”

Problem - General Foundry

- General Foundry has been given 14 weeks instead of 16 weeks toinstall the new equipment
- The critical path for the project is 15 weeks
- The normal and crash times and costs are shown in Table 13.9
- Crash costs are assumed to be linear
- Crashing activities *B* and *A* will shorten the completion time to 14 but it creates a second critical path

General Foundry Example

- Normal and crash data for General Foundry

ACTIVITY	TIME (WEEKS) NORMAL	TIME (WEEKS) CRASH	COST ($) NORMAL	COST ($) CRASH	CRASH COST PER WEEK ($)	CRITICAL PAT H?
A	2	1	22,000	23,000	1,000	Yes
B	3	1	30,000	34,000	2,000	No
C	2	1	26,000	27,000	1,000	Yes
D	4	3	48,000	49,000	1,000	No
E	4	2	56,000	58,000	1,000	Yes
F	3	2	30,000	30,500	500	No
G	5	2	80,000	86,000	2,000	Yes
H	2	1	16,000	19,000	3,000	Yes

Table 13.9

Crash and Normal Times and Costs for Activity B